PERFORMANCE ANALYSIS OF MULTI-RATE CAPABLE RANDOM ACCESS MAC PROTOCOLS IN WIRELESS MULTI-HOP NETWORKS

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ABSTRACT

In this paper, we study an ad hoc wireless network system in which multiple access wireless channels are shared in accordance with a random access (ALOHA type) MAC protocol. We assume that a modulation/coding scheme (MCS) can be selected from a list of available such structures, each of which is associated with a transmission data rate and an acceptable SINR level that yields a prescribed BER value. The use of such different MCSs impacts the achievable value of the system’s spatial reuse factor level and of the realized end-to-end lengths of flow paths. Consequently, as the traffic loading level increases, the use of such different schemes yields different throughput capacity levels. We present mathematical models that enable the selection of the MCS that yields the highest level of throughput performance. For the multi-hop network system, we demonstrate that the link range and the employed MCS can be jointly selected to yield the highest throughput capacity level. We show that for the system under consideration it is generally preferable for a node to use a high data rate MCS while jointly selecting its neighbours from a usually reasonably short range level.

I. INTRODUCTION

The rapid development and tremendous success of wireless LANs over the past few years have nourished the advancement in large scale multi-hop ad hoc networks. To support multimedia flows over resource limited wireless network systems, in particular when multi-hop network operations are involved, it has become necessary to design systems that operate at higher data rates. Such operations are often implemented by using more efficient modulation/coding schemes (MCSs). Across each communications link, the selected operation is subjected to the fundamental tradeoff between the employed data rate (as dictated by the selected MCS) and the ensuing minimum required SINR (Signal and Interference to Noise Ratio) level required to achieve a targeted BER level. To demonstrate the impact of such a design tradeoff, we consider a network in which wireless communications links operate at a relatively high data rate level. Since a high data rate transmission generally requires a higher SINR value at the receiver to yield successful packet reception, a corresponding lower level of interference is tolerated. Thus, when nodes proceed to transmit at higher data rates, one expects the realized average value of the Spatial Reuse Factor (SRF) to assume lower values. Furthermore, from network layer point of view, since a high data rate transmission can require the use of shorter link level maximum transmissions ranges, a flow that is transported may have to be directed along a route that contains a large number of hops. Such increased path length will serve to reduce the gains attained by using higher transmit data rates. Hence, by increasing the nodal transmit rate, one is not necessarily attaining an overall upgraded network wide throughput rate and an improved delay-throughput performance behavior.

Clearly, the results characterizing such a tradeoff, and the ensuing optimal design of the system through the use of an adaptive rate operation depends on the underlying system structure, including the underlying medium access control (MAC) protocol. It clearly impacts the amount of throughput capacity gain attainable through the proper selection of the data rate. In this study, we assume the network system to employ a random access (Aloha type) MAC scheme. Such a contention based operation is universally employed by many wireless network systems that require a robust and fully distributed scheduling scheme. Its operations are simpler than the extended contention algorithms that employ channel sensing operations; they thus avoid the need to deal with carrier sensing range optimization that is often considered when studying multi-rate CSMA (Carrier Sense Multiple Access)-type network systems ([11]-[13]). In this paper, we model and characterize the behavior of a multi-rate multiple access Aloha wireless network under both single-hop and multi-hop configurations. Mathematical formulas expressing the network’s MAC throughput performance are derived, based on random topologies. We also study the impact of multi-rate multi-hop forwarding operations on the end-to-end network’s realized throughput performance. For this purpose, we use the bit-distance product metric identified as the transport throughput. For illustration purposes, we use the MCS employed by IEEE 802.11a systems. Our results provide important guidelines for the design of rate adaptation algorithms. Furthermore, they provide characterizations that can be used in choosing the channel access strategy, and in performing route selections in a multi-hop wireless network.

We note that no published work to date has yet explored the performance tradeoffs involving the selection of the MCS and the ensuing throughput performance attained by the underlying multi-hop ad hoc wireless network system, as particularly impacted by the attainable SRF level. The purpose of this paper is to carry out such an investigation for a multi-hop ad hoc wireless network system under the use of a random access (pure ALOHA type) MAC scheme. The organization of the rest of this paper is as follows. Section II overviewed related works. In section III, we specify the system model and derive mathematical formulas for calculating the (link layer) throughput of a multi-rate Aloha system. Performance results are exhibited in section IV. The impact of multi-hop forwarding operations on the network’s end-to-end throughput performance is discussed in section V.

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In the context of multi-rate wireless networks, we note studies that involve the use of rate adaptations for Wireless LANs ([1]-[5],[8]-[10]) or for ad hoc networks ([6]-[7]). Rate adaptation algorithms generally aim to adapt fast and correctly to the features of a time varying channel. Rate adaptation algorithms can be further classified into two categories ([10]): closed-loop and open-loop. These works have generally not incorporated the multiple access nature of wireless networks and the ensuing co-channel interference conditions that can induce packet collisions. In [9], [10], collision aware algorithms are proposed in the context of Wireless LANs, not involving multi-hop network layouts. In [11]-[13], the tradeoff between spatial reuse and capture failure is identified and an optimal carrier sensing threshold is calculated to maximize the spatial reuse factor achievable in an 802.11-based ad hoc network operation, when regular topologies are assumed. In [13], the authors have considered variable transmission ranges and receiver sensitivities, associated with different modulation coding schemes, including the impact of multi-hop forwarding. In [19], a closed form expression is obtained for the product of the number of successful transmissions per unit space and the link transmission range for a large multi-hop network system using Slotted ALOHA MAC operation. In this paper, the authors assume the distribution of nodal locations to be governed by a Poisson distribution, while the transmit power level is taken to assume an exponential distribution.

II. SYSTEM MODEL

In this section, we aim to derive the (link layer) aggregate throughput expression for general network topologies and MCSs. Stations can form communication links with their direct neighbors, when feasible, for the purpose of routing their packets to their corresponding destinations in a multi-hop fashion. For performance tradeoff illustrative purposes, we assume that the same selected MCS is used by all nodes across all links for a prescribed duration of operations. During such a period, the network is characterized by observed traffic loading and nodal entity location patterns and distributions. We assume that n stations are randomly distributed over a two dimensional area of operations (a x a). Stations (also identified as nodes) are assumed to use half-duplex radios. A SINR-based interference model is used. For our analytical derivation, we assume a power law path loss model, i.e., $G_{xy} = \eta R_{xy}^{-\alpha}$ where $G_{xy}$ and $R_{xy}$ denote the link gain and distance from node x to y, $\eta$ the path loss constant, and $\alpha$ the attenuation factor, $\alpha > 0$.

A. ALOHA MAC

We consider a system which employs a pure (unslotted) Aloha scheme as its MAC protocol. Each station is assumed to contain a single packet buffer. Note that the use of a corresponding slotted MAC algorithm would have induced the following requirements: First, one would have to maintain a synchronized clock at the various nodes, which is generally a demanding task when an ad hoc multi-hop network system is involved. Second, we intend to expand our model to allow for variable rate operations; clearly, it is then simpler to implement a system that operates in an asynchronous manner. As shown in Fig. 1, an idle station generates a new packet after a random idle period that is exponentially distributed with an average of $1/\lambda$, sec. Packets are of constant length (consisting of b bytes of MAC layer frame data). Upon a packet arrival, a station will proceed to transmit it immediately across its designated link. If packet transmission fails, the station will proceed to retransmit the packet after a random delay; the latter is assumed to follow an exponential distribution with mean $1/\lambda$. To characterize the performance of such a network when loaded by a wide variety of spatially distributed flows, having a multitude of source-destination pairs, we assume that a station selects the intended receiver for a transmitted packet at random from among its link layer neighbors.

$$P_s(r_c) = \Pr\{\text{node } y \text{ is idle during transmission } x \rightarrow y\} \cdot \Pr\{\text{SINR at node } y \geq \gamma\}$$

where $\Pr\{\text{node } y \text{ is idle during transmission } x \rightarrow y\}$ is the probability of y being idle at the start of transmission x→y and can be written by:

$$P_s(r_c) = \frac{P}{N + \sum_{t=1}^{T} PG_{\gamma}}$$

where P denotes the transmission power, N denotes the thermal noise power, and index t denotes the t-th node that transmits during transmission x→y. The first term of the product expression in Eq. (1) can be represented as the probability of y being idle at the start of transmission x→y and remaining idle for T seconds. We thus obtain:

$$\Pr\{\text{node } y \text{ is idle during the transmission } x \rightarrow y\} = \frac{T}{T + B} e^{-\lambda T}$$

where idle period $T = \frac{1}{\lambda}$, and busy period $B = T$.

We note the function that describes the distribution of the number of interferers during transmission x→y is geometrically distributed with parameter $(1-P_c)$. $P_c$ is the probability that a single node stays idle during transmission x→y. Hence the second probability term in Eq. (1) can be written by:

$$P_s(r_c) = \frac{P}{N + \sum_{t=1}^{T} PG_{\gamma}}$$

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\[
\Pr\{k \text{ nodes transmit during transmission } x \rightarrow y\} = \binom{n-2}{k} (1 - P_e)^k P_e^{n-2-k}, k = 1, 2, ..., n - 2. \tag{3}
\]

For the third probability term that states the SINR requirement, we first note that the \(G_{xy}\) and \(G_y\) terms are random variables that are link distance sensitive. The \(G_{xy} = t = 1, ..., k\) terms are assumed to be independent and identically distributed. From [15], the probability density function (p.d.f.) characterizing the distance \(R\) between two nodes that are uniformly distributed across a square area (a x a) is given by

\[
f_g(r) = \frac{4r}{a^2} f_o(r) \tag{5}
\]

where

\[
f_o(r) = \begin{cases} \frac{\pi}{2} a^2 - 2ar + \frac{1}{2} r^2, & \text{for } 0 \leq r \leq a \\ a^2 \sin^{-1} a - 2a \sqrt{r^2 - a^2} - a^2 - a^2 \cos^{-1} a - \frac{1}{2} r^2, & \text{for } a \leq r \leq \sqrt{2a} \\ 0, & \text{elsewhere} \end{cases}
\]

Through a change of variable operation from \(R_{xy}\) to \(G_{xy}\) based on the power law path loss model, the p.d.f. of \(G_{xy}\) can be derived and is related to Eq. (5) as follows.

\[
f_c(g) = \frac{dR(g)}{dg} f_o(\sqrt{g})^{-\frac{1}{a}} \tag{6}
\]

We readily calculate the SINR requirement probability term for the cases where \(k\) is equal to 0 or 1 as long as the density function of \(G_{xy}\) is known. When \(k = 1\):

\[
\Pr(G_x \geq \frac{\gamma N}{P} + \gamma G_y) = \int_{0}^{\infty} \delta \left(\frac{\gamma N}{P} + \gamma G_y\right) f_o(g)dg \tag{7}
\]

For the cases where \(k\) is larger than 1, we approximate the probability of capture by considering the major interfering signal as the dominant component. This simple approach reduces the interference modeling complexity and allows us to proceed with detailed investigation of the underlying multi-rate multi-access problem. Such approach has been shown in [18] to yield accurate results, in comparing the analytical results with simulation evaluations, when the loading level is not overly high. It is compatible with the observation made in [14] noting that the received power level measured at a node due to reception from the nearest interferer is of the same order as the total interference contributed by signals emitted across the entire network. The probability of capture is then given by:

\[
\Pr(G_n \geq \frac{\gamma N}{P} + \gamma \sum_{i=1}^{n} G_y) \approx \Pr(G_n \geq \frac{\gamma N}{P} + \gamma G_{(n,k)}) \tag{8}
\]

where \(G_{(n,k)}\) denotes a random variable that has the same distribution as the largest gain of the \(k\) i.i.d. random variables \(\{G_{xy}, t = 1, ..., k\}, G_{(n,k)} = \max\{G_{xy}, t = 1, ..., k\}\). Its probability density function is noted to be:

\[
f_{c,(n,k)}(g_{(n,k)}) = k f_{c,g}(g_{(n,k)}) \int F_c(g_{(n,k)})^{n-k} \tag{9}
\]

Combining Eqs. (1)-(9), we calculate \(P_r(r_c)\) accordingly. The aggregate (link layer) throughput \(S\) is given by

\[
S(r_c) = G \cdot P_r(r_c) \tag{10}
\]

\[
G = nb\frac{T}{I + B} \left[ \text{bit second} \right] \tag{11}
\]

where \(G\) is the network carried load. In the next section, we apply the presented model and evaluate the analytical results of the multi-rate Aloha MAC throughput performance under different network configurations.

### III. AGGREGATE LINK LAYER THROUGHPUT PERFORMANCE IN MULTI-RATE WIRELESS NETWORKS

#### A. Single Hop Wireless Networks

In a single hop network, any two nodes located in the area of operation are direct neighbors of each other. Therefore, the link gain between the transmitter and the receiver \(G_y\) has the same p.d.f. as \(G_{xy}\) (Eqs. (5)-(6)). We use the MCSs provided by IEEE 802.11a. The parameters are listed in Table 1, under a targeted BER value that is equal to \(10^{-5}\) ([16]). Note that we also account for the PHY and MAC header overhead values. The latter levels have an impact on the underlying performance. The PLCP preamble plus SIG duration, denoted as \(T_0\), and the data rate dependent overhead length induced by the PHY and MAC header values, denoted as \(b_0\), are each equal to 20 μs and 246 bits, respectively, for the IEEE 802.11a protocol. The packet transmission time \(T\) is thus given by

\[
T = T_0 + (b + b_0)/r_c.
\]

Note that the \(T_0\) term induces higher channel inefficiency and higher probability of capture failure for transmissions that are selected to operate at higher data rate level. The resulting link throughput vs. load performance is shown in Fig. 2. In general, the attained throughput performance level is higher when higher data rate values are used.

#### B. Multi-Hop Wireless Networks

We use the analytical expressions derived above to evaluate the performance behavior of a multi hop network. To illustrate and compare the behavior of this system, we assume in the following evaluations that there are 150 nodes uniformly distributed in a 1000m x 1000m square area. The transmit power level used by each node is fixed at 15 dBm. Note that the SINR threshold requirements at different data rate levels are different, so the corresponding maximum transmission ranges (when there are no interfering transmissions) also differ. The spatial reuse gain is expected to become more dominant compared to that in single-hop
scenarios, since spatial reuse within single hop scope is in nature more difficult to achieve. Two rules for selecting forwarding nodes are used in evaluating link throughput performances.

<table>
<thead>
<tr>
<th>Rate (Mbps)</th>
<th>6</th>
<th>9</th>
<th>12</th>
<th>18</th>
<th>24</th>
<th>36</th>
<th>48</th>
<th>54</th>
</tr>
</thead>
<tbody>
<tr>
<td>f (dB)</td>
<td>6.02</td>
<td>7.78</td>
<td>9.03</td>
<td>10.79</td>
<td>17.04</td>
<td>18.8</td>
<td>24.05</td>
<td>24.56</td>
</tr>
</tbody>
</table>

Table 1: Data rate – SINR threshold table with target BER = 10^-5.

![Fig. 2](image2.png)

Fig. 2 Throughput vs load performance in single hop networks under data rate levels provided by IEEE 802.11a.

1) Choose forwarding (next-hop) nodes from the neighbor list that is constructed based on the underlying operating data rate.

Under this rule, each node that is at a distance from the underlying node that is not exceeding the maximum transmission range is identified as a neighboring node. Assume nodes x and y to be the transmitter and receiver nodes, respectively, and all nodes to use a MCS that operates at rate r_c. The link distance variable (between nodes x and y) is denoted as R_{xy}. Its p.d.f. is given by a uniform distribution f_{R_{xy}}(r) = 2r/(R_{max}^2), 0 \leq r \leq R_{max}, where R_{max} denotes the maximum transmission range when the data rate is equal to r_c. This is then used to calculate the probability density function of G_{xy} by using Eq. (6) to obtain results for the network’s throughput performance. The resulting aggregate network link layer throughput performance (expressing the sum total of throughput rates realized across all network links) is depicted vs. the offered network load in Fig. 3(a). We find that an operation at a higher data rate MCS yields a significantly higher aggregate link layer throughput. Note that the interference at an identified receiver does not change as the underlying data rate and forwarding rule is changed. Hence, when a node selects its forwarding neighbour to reside closer to itself, its signal is received at a higher SINR level. However, this level may not be sufficient to ensure acceptable reception when operating at a higher data rate. Thus, it is not immediately obvious whether an operation at a higher data rate would lead to improved throughput performance.

2) Choose the forwarding nodes from the neighbor list constructed based on the highest provided data rate.

Under this rule, we use the maximum distance range that is calculated based on the highest data rate MCS to define node’s neighbors. In this case, f_{R_{xy}}(r) = 2r/R_{max}^2, 0 \leq r \leq R_{max}, where R_{max} denotes the maximum transmission range associated with the highest available data rate. We observe from Fig. 3(b), that the attained aggregate link layer throughput is higher than that achieved by using forwarding rule 1. Furthermore, the relative improvement increases as the data rate level is decreased (in comparison with the previous performance under the same MCS). The optimal data rate MCS that maximizes system link layer throughput performance can be dependent on the network loading level. In Fig. 3(b), for the displayed offered load range, we observe the MCS that operates at 18Mbps data rate to yield best performance. We thus find that forwarding rules play an important role in impacting the aggregate network link layer throughput performance.

![Fig. 3](image3.png)

Fig. 3 Link throughput vs load performance in multi-hop networks under data rate levels provided by IEEE 802.11a using (a) configuration 1, (b) configuration 2.

IV. TRANSPORT THROUGHPUT PERFORMANCE IN MULTI-HOP NETWORKS

We define the sum of the products of throughput values (in bits per second) and distances over which those bits are transported as the transport throughput; it is denoted as S_t. This bit-distance product has been used as an indicator of a network’s capability of transporting data from one end to the other ([17]). Assuming an operation under which the average path distance value is determined by the underlying nature of the spatial distribution of user flows, we proceed to use the transport throughput as an indicator of the system’s end-to-end throughput performance. Suppose that a transmitter (operated by a relay or source node) can always find a next hop node in a desired location. We may then vary the hop distance as a parameter to see its effect on the transport throughput level, using different transmission data rate levels. If the hop distance range is set equal to a very low value, there will be hardly any data bits ‘transported’. On the other hand, if we keep increasing the hop distance, capture failures will eventually dominate and the realized throughput level will be extremely low. Therefore, we note that there exists an optimal hop distance value that yields a maximum value for the realized transport throughput level. The optimal hop distance is parameterized by both the transmission data rate and the network carried load. We
exhibit this hop distance vs. transport throughput performance tradeoff behavior through the results shown in Figs. 4-6. In Fig. 4(a), each curve (numbered from 1 to 9) represents the use of a hop distance that starts from a minimum value of 20 m and increases at a 10 m step size, using a MCS operating at 6 Mbps. The transport throughput is plotted vs. the carried transport load. At very light transport loading levels, the transport throughput attained by selecting the minimum hop distance value is observed to outperform others, though not by a significant margin. As the transport loading level increases, forwarding rules that employ short hop distance levels start to generate a high internal traffic rate level (noting that messages now tend to travel along routes that contain more hops). Consequently, a lower probability of signal capture is realized by a node at its intended link’s receiver. It is thus noted that as the transport loading increases, it becomes more efficient to use longer hop distances so that the route’s path length is reduced. In Fig. 4(b), the same setup as that for Fig. 4(a) is used, except for a different MCS operating at a data rate of 54 Mbps. We find from Fig. 4(b) that, the use of a shorter hop range leads to better transport throughput performance, when operating under a loading rate level in the displayed range. We note that the optimal hop range level is even smaller than 20m when the network uses the 54 Mbps MCS. Clearly, in a specific operational configuration, one may not be able to identify neighboring nodes that are located at such a close range.

In Fig. 5, we depict the transport throughput performance vs. the selected hop distance level, under the use of a fixed 18 Mbps data rate, under transport loading levels that range from $10^9$ to $10^{10}$ bps*m. The resulting curves (numbered from 1 to 10 to correspond to the different loading rates) indicate that the optimal network transport loading level that optimizes the transport throughput performance is a function of the selected forwarding hop distance level. The results presented demonstrate the impact induced by the joint use of the MCS (and its data rate), the hop range, and the carried end-to-end network load on the realized transport throughput performance.

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