Analysis of an FDDI network supporting stations with single-packet buffers

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Abstract

We present exact analysis of a nonsymmetric multiple-priority token ring network with single packet buffers under a timed-token protocol, similar to that employed for medium access control of the Fiber Distributed Data Interface (FDDI) network. The single-packet buffer model is employed for describing the queueing operation characterizing typical user terminals. An iterative procedure is used to compute the limiting state distribution of the embedded Markov chains representing the system state process. We obtain the distributions of the token rotation time, the normalized throughput, and the mean packet waiting time.

By using a counting process representing the numbers of transmitted and deferred packets in a token rotation cycle, we present a simplified analysis of a symmetric network containing a large number of stations. We illustrate the application of the analytic approach through numerical examples representing FDDI network systems operating under various traffic loading conditions.

Keywords: FDDI; Single-packet buffers; Timed-token protocol; Token ring network; Token rotation time; Packet waiting time; Medium access control

1. Introduction

Timed-token protocols (TTP) [5,16] have witnessed their widespread acceptance in the area of medium access control (MAC) for local area networks. Under the ANSI-based timed-token ring standard, FDDI [6,11,12], and the IEEE-defined timed-token bus standard, 802.4 [1], two classes of service are provided. (1) A synchronous class of service is intended for applications that require guaranteed bandwidth and response time, such as real-time packet voice/video or control signals. (2) An asynchronous class of service is available for multiple priority levels of traffic, such as file transfer or bursty data packets, that share the available system bandwidth dynamically.

Performance of the FDDI network has been examined in various aspects. Jain [8] presented the impact of the target token rotation time (TTRT) on the throughput and waiting times in an FDDI network. A TTRT value

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of 8 msec was reported to provide a good performance over a wide range of configurations and workloads. Dykeman and Bux [3] conducted approximate throughput analyses of FDDI under general traffic configurations. Pang and Tobagi [10] presented exact throughput analysis of the IEEE 802.4 timed-token bus operating under heavy load.

Discrete-time analyses have been previously conducted for ring networks serving stations with single packet buffers. Tsai and Rubin [15] studied the queueing behavior of symmetric token ring networks governed by the IEEE 802.5 token ring standard. Takagi [14] investigated the effects of the token rotation time on the delay-throughput performance of a symmetric single buffer system operating under a timed-token protocol. In the latter, a Gauss elimination method is used to calculate the limiting state distribution of the embedded Markov chain for a network connecting a small number of stations.

The inherent chaining property in a cyclic service (polling) system is revealed when the system is examined during token visits to the stations. By exploiting this chaining property, Ferguson and Aminetzah [4] presented exact analyses of polling systems with exhaustive and gated services, while Konheim [9] analyzed approximately a polling system with limited service. Note, however, that under the FDDI TTF, dynamic time limits are imposed on the token holding duration permitted at each station.

In this paper, we analyze a nonsymmetric token ring network supporting stations with single packet buffers, under a timed-token protocol similar to that employed by the FDDI medium access control. For an FDDI network serving a moderate number of stations with small buffer capacity, this analysis provides an efficient approach for evaluating the queueing behavior of the network system. For a nonsymmetric FDDI network in which a host computer is connected with a cluster of lightly loaded terminals, this model of single-packet buffered stations is suitable for analyzing the terminal cluster, while an M/G/1/N vacation model [13] is adequate for the analysis of the host computer. In Section 5, we demonstrate the effectiveness of combining the latter two models.

The analysis carried out in this paper extends the methods used in [15,14]. We characterize a set of discrete-time Markov chains at the instants of token arrivals at the stations. An efficient iterative procedure is used to compute the limiting state distributions of the embedded Markov chains, without inverting the transition probability matrix. We present a simplified analysis of a symmetric network containing a large number of stations by using counting processes representing the numbers of transmitted and deferred packets in a token rotation cycle. We investigate the performance of FDDI network systems loaded by various characteristic traffic configurations.

In Section 2, we describe the modeling of ring networks operating under the timed-token protocol. In Section 3, we carry out the discrete-time Markov chain analysis. An iterative procedure for the calculation of the limiting state distributions of the embedded Markov chains is presented. We obtain the distributions of the token rotation time, the normalized throughput, and the mean waiting time. In Section 4, we introduce a simplified method employing a counting process and the recursive computations of its associated transition probabilities to analyze a symmetric network containing a large number of stations. In Section 5, we illustrate the application of this analytic approach to the performance evaluation of common configurations of FDDI token ring systems.

2. System description

We consider a nonsymmetric token ring network in which the medium is shared in accordance with a timed-token protocol such as the FDDI Medium Access Control (MAC) scheme. The network consists of K stations (numbered as 0, 1, . . . , K − 1), each of which has a buffer that can accommodate only a single packet. A token rotates around the ring and visits station-0, station-1, . . . , station-(K − 1) cyclically.

A target token rotation time (TTRT) is selected during ring initialization. Multiple priority levels of asynchronous service are supported and distinguished by the use of distinct timing thresholds: $T_{pri,i}$ for asynchronous
priority $i$ service, $i > 1$, where $T_{pri-i} = T_{TRT}$, $T_{pri-i} < T_{pri-j}$, $i > j > 1$; packets which receive synchronous service are said to be of priority-0. (Priority-$i$ packets assume lower priority than priority-$j$ packets, for $j < i$.)

To simplify the model, we assume that the traffic stream at an individual station is accommodated by a single priority class of service. We use $P_k$ to denote the priority class of service at station-$k$, $k = 0, 1, \ldots, K - 1$. The timing threshold at station-$k$ is then equal to $T_{pri-P_k}$.

The token rotation time ($TRT$) at a station is equal to the time between successive token arrivals at the station. We observe two classes of stations according to the class of service provided: synchronous and asynchronous stations.

1. When the token arrives at a synchronous station with a packet queued (for transmission), the station is allowed to transmit this packet, independently of the underlying token rotation time.

2. For an asynchronous station, the following timed-token protocol is used. When the token arrives at a station that has a queued packet, the station is allowed to transmit the packet if the current token rotation time does not exceed its timing threshold (i.e., the token arrives early: $TRT < T_{pri-P_k}$ for station-$k$); the token then departs from this station after its queued packet is transmitted. The token departs immediately from a station when the station has no packet queued for transmission or when it has arrived late at the station (i.e., $TRT \geq T_{pri-P_k}$ for station-$k$).

The following (nonsymmetric) station traffic model is assumed: Station-$k$, $k = 0, 1, \ldots, K - 1$, takes an exponentially distributed interval of time with mean $1/\lambda_k$ to generate a new packet after transmission of the previous packet. The packet transmission time at each station is set equal to 1 (slot).

The token walk time across the ring (ring latency) is assumed to be governed by $r(i) = P(r_0 + i)$, $i = 0, 1, \ldots, R_{max}$, $R_{max} < \infty$, where $r_0$ is a minimum (constant) walk time component and $P$ is a probability function. Token walk times between successive stations are assumed to be statistically independent. The mean token walk time around the ring is denoted as $R$.

The equivalent timing threshold associated with station-$k$, denoted as $T_k$, $k = 0, 1, \ldots, K - 1$, is defined by $T_k = T_{pri-P_k} - r_0$. To avoid the effect of asynchronous overrun at a station, we assume that $T_k$ is an exact multiple of the slot time, $k = 0, 1, \ldots, K - 1$.

The operation described by the model presented above is similar to that exhibited by an FDDI network under the corresponding set-ups of the timing thresholds for synchronous and asynchronous services. When the token arrives at a synchronous station that has a queued packet, the station can always transmit this packet. Thus, the packet transmission time reflects the corresponding synchronous FDDI bandwidth allocation to a station; each synchronous station is allocated a fraction of $T_{TRT}$ for its token holding time. Upon token arrival at an asynchronous station with a queued packet, a packet will be transmitted only if the token rotation time is within the timing threshold of the station. Since each station is assumed to have a single packet buffer, a packet queued at a station experiences no local queueing delay, but rather a token latency delay (in waiting for the arrival of a usable token). While packets arriving at a synchronous station are served at each token arrival, packets queued at an asynchronous station are provided with dynamic time-limited service based on the (global) token rotation time.

The assumptions of a single-packet buffer and a single priority level for an FDDI station can be justified in considering the following practical applications:

1. The FDDI network is used to interconnect a number of users and a video server in a video on demand system [21]. The video streams coming from the video server to the users constitute the majority of the overall system traffic since the command issued by a user (e.g., the “start playing” command in a single packet) usually demands the transmissions of a large number of packets by the video server (e.g., a 10-minute video program compressed by the 1.5 Mbits/s MPEG-1 [7] standard may consist of up to 112 Mbytes). Despite receiving the large number of packets from the video server, the FDDI station attached to a user usually has no more than a queued (command) packet, and its queueing behavior conforms to the assumption of a single-packet buffer.

2. The asynchronous service of FDDI is known to provide fair access to the medium when the stations have the same (single) priority level [8,5]. In practice, a single priority level of asynchronous service is commonly
used for the FDDI stations. On the other hand, when the synchronous traffic at an FDDI station is heavy, the
time spent in synchronous service during each token visit is equal to the corresponding (constant) synchronous
bandwidth, which can then be incorporated in the overall token walk time. Virtually all FDDI product vendors
are offering just a single priority asynchronous service; as a result, most users of FDDI networks employ a
single priority asynchronous FDDI service. It is therefore adequate to assume that an FDDI station is provided
with a single priority level of asynchronous service to work out efficient analytic methods proposed in this
paper.

3. The embedded Markov chain

3.1. The embedded state processes

We examine the system at the instants of token arrivals at the stations. Let \( \tau_k^n \) denote the instant of the \( n \)th
token arrival at station-\( k \), \( k = 0, 1, \ldots, K - 1 \), \( n \geq 0 \). The state of station-\( k \) at time \( \tau_k^n \) is defined as follows:
- \( s_k^n = 0 \) (buffer empty) if station-\( k \) has no packet queued,
- \( s_k^n = 1 \) (packet transmitted) if station-\( k \) has a packet queued that will be transmitted,
- \( s_k^n = 2 \) (packet deferred) if station-\( k \) has a packet queued that cannot be transmitted.

Note that for a synchronous station, \( s_k^n \neq 2 \), since a synchronous station with a packet queued can always
transmit a packet when the token arrives at the station.

We define the \( n \)th token rotation cycle with respect to station-\( k \) to be the \( n \)th token interarrival period at
station-\( k \), \( \{ \tau_k^n, \tau_k^{n+1} \} \), \( n \geq 0 \). The corresponding token rotation (cycle) time is denoted as \( T_k^n \). Let \( M_k^n \) denote
the overall number of packets transmitted at all the \( K \) stations in the system during the token rotation cycle
\( \{ \tau_k^n, \tau_k^{n+1} \} \). Note that \( M_k^n = \sum_{i=k+1}^{K-1} i(s_i^n = 1) + \sum_{i=0}^{k-1} i(s_i^{n+1} = 1) \) and \( T_k^n = \tau_k^{n+1} - \tau_k^n \).

The states of the stations in the system during the \( n \)th token rotation cycle with respect to station-\( k \),
\( k = 0, 1, \ldots, K - 1 \), are described by a \( K \)-tuple vector \( \sigma_k = (s_{K-1}^n, s_{K-2}^n, \ldots, s_0^n) \). For each \( k \in \{0, 1, \ldots, K - 1\} \), the state process \( \{ \sigma_k \} \) constructs a discrete-time Markov chain over the state
space \( \mathcal{A} \):

\[
\mathcal{A} = \{(u_0, u_1, \ldots, u_{K-1}) \mid u_j = 0, 1, 2, j = 0, 1, \ldots, K - 1\}.
\]

We note that a total of \( K \) discrete-time Markov chains are defined: \( \sigma_0 \), \( \sigma_1 \), \ldots, \( \sigma_{K-1} \). The number of
the overall states in \( \mathcal{A} \), denoted as \( |\mathcal{A}| \), is noted to be equal to \( 3^K \).

3.2. The balance equations

The limiting state distributions are defined as

\[
\pi_k(s_k) = \lim_{n \to \infty} P(S_k^n = s_k), \quad s_k \in \mathcal{A}, \quad k = 0, 1, \ldots, K - 1,
\]

where \( s_k \) is defined by \( s_k = (s_{K-1}, s_{K-2}, \ldots, s_0) \).

For each \( k \in \{0, 1, \ldots, K - 1\} \), the underlying Markov chain \( \sigma_k \) has a finite state space and is noted to be
irreducible and aperiodic. Therefore a unique limiting (steady-state) distribution exists.

The steady-state probabilities satisfy the following set of balance equations:

\[
\pi_k(s_{k+1}, \ldots, s_{K-1}, s_0, \ldots, s_{k-1}, s_k) = \sum_{u_k=0,1,2} \bar{h}_{s_{k+1}}(u_k) \pi_k(s, s_{k+1}, \ldots, s_{K-1}, s_0, \ldots, s_{k-1}, s_k),
\]

\[
(s_{k+1}, \ldots, s_{K-1}, s_0, \ldots, s_{k-1}, s_k) \in \mathcal{A}, \quad k = 0, 1, \ldots, K - 1;
\]

and
\[ \mathbf{H}(\mathbf{v}) = \begin{bmatrix} f^k(\mathbf{v}) & \ldots & f^k(\mathbf{v}) \\ \vdots & \ddots & \vdots \\ f^k(\mathbf{v}) & \ldots & f^k(\mathbf{v}) \end{bmatrix} \]

We summarize the evaluation of $\mathbf{H}(\mathbf{v})$ in the following matrix:

\[ \begin{bmatrix} f^k(\mathbf{v}) & \ldots & f^k(\mathbf{v}) \\ \vdots & \ddots & \vdots \\ f^k(\mathbf{v}) & \ldots & f^k(\mathbf{v}) \end{bmatrix} \]

The elements of the matrix $\mathbf{H}(\mathbf{v})$ are given by

\[ f^k(\mathbf{v}) = \sum_{i=0}^{R_{\text{max}}} e^{-A_i(v+r_{i+1})} \cdot r(i), \quad v = 0, 1, \ldots, K - 1, \]

\[ \tilde{g}^k_i(\mathbf{v}) = \begin{cases} \left[ T_k - i - 1 - v, \ldots, R_{\text{max}} \right] & i = 0, 1, \\ v = 0, 1, \ldots, [T_k - i, K] - 1, \\ 0, & v = T_k - i, \ldots, K - 1; \end{cases} \]

\[ \tilde{f}^k_i(\mathbf{v}) = \begin{cases} \left[ T_k - v - 1 - i, \ldots, R_{\text{max}} \right] & v = 0, 1, \ldots, [T_k, K] - 1, \\ 0, & v = T_k, \ldots, K - 1, \end{cases} \]

where $\lfloor \cdot \rfloor$ denotes the minimum of the arguments.

We note the following: (1) In writing Eq. (3), we use $w_{\mathbf{R}}(s_0, s_1, \ldots, s_{K-1}) = \pi_0(s_0) \pi_1(s_1) \cdots \pi_{K-1}(s_{K-1})$ for notational simplicity. (2) In Eq. (5), we adopt the conventional notation by defining $\pi_{k+1} = \pi_k^{0+1}$.

### 3.3. Iterative calculations of the limiting state distributions

By examining the balance equations (see Eq. (3)), we observe that the state transition matrix is sparse due to the chaining property of the cyclic service system. The matrix defined in Eq. (6) is noted to have at most 8 positive entries. Let $N_s$ denote the total number of transitions between states $\pi_0$ and $\pi_{K+1}$. We observe that $N_s \leq 8 \cdot 3^K - 1$. Let $N_{sf}$ denote the number of state transitions in a fully connected arrangement over state space $\mathcal{A}$. We know that $N_{sf} = |A|^2 = 3^{2K}$. We thus obtain the ratio of $N_s$ to $N_{sf}$, representing the index of sparsity associated with the state transition matrix, to be bounded by $N_s/N_{sf} \leq 8/3^{K+1}$. This property of sparsity suggests
that the limiting state distributions can be evaluated very effectively by iterative computations instead of matrix inversion, as $K$, the number of stations in the system, increases. We therefore present the following iteration procedure, which proceeds in a round-robin fashion, for the calculation of the limiting state distributions.

**Procedure 1. Iterative computations of $\{\pi_k(s_k)\}$.**

(Initialization) $i \leftarrow 0$;

$\pi^{(0)}_0(\emptyset) \leftarrow 1; \quad \pi^{(0)}_0(s_0) \leftarrow 0, \forall s_0 \in A, s_0 \neq \emptyset$.

(Iteration) (successive computations: $\pi^{(i)}_0(s_0) \rightarrow \pi^{(i)}_1(s_1) \rightarrow \cdots \rightarrow \pi^{(i)}_{K-1}(s_{K-1}) \rightarrow \pi^{(i+1)}_0(s_0)$)

- For $k = 0, 1, \ldots, K - 2$, calculate $\{\pi^{(i)}_{k+1}(s_{k+1})\}$, given $\{\pi^{(i)}_k(s_k)\}$, according to Eq. (3).
- $i \leftarrow i + 1$;
- Calculate $\{\pi^{(i)}_0(s_0)\}$, given $\{\pi^{(i-1)}_{K-1}(s_{K-1})\}$, according to Eq. (3).

(convergence check) if $\sum_{i \in I} |\pi^{(0)}_0(s_0) - \pi^{(i)}_0(s_0)| > \epsilon$, repeat (iteration)

else stop; $\{\pi^{(i)}_0(s_0), \pi^{(i-1)}_1(s_1), \ldots, \pi^{(i-1)}_{K-1}(s_{K-1})\}$ are the limiting state distributions.

3.4. Statistics for performance evaluation

Let $M_k(m)$, $m = 0, 1, \ldots, K - 1$, denote the probability that $m$ packets are transmitted during a token rotation cycle with respect to station-$k$, $k = 0, 1, \ldots, K - 1$. We have

$$M_k(m) = \sum_{m=0}^{m=K-1} \pi_k(s_k), \quad m = 0, 1, \ldots, K - 1. \quad (10)$$

Let $\bar{M}_k$ denote the average number of packets transmitted during a token rotation cycle with respect to station-$k$. Note that for a cyclic service system, $\bar{M}_k$ is invariant with respect to $k$, i.e., $\bar{M}_k = M_i = \bar{M}$, $i \neq k$.

We use $\bar{G}_k$ to denote the average number of packets transmitted by station-$k$ during a token rotation cycle with respect to station-$k$. $\bar{G}_k$ can be calculated as follows:

$$\bar{G}_k = \sum_{k,0,1,2,\ldots,K-1} \pi_k(1, s_{k+1}, \ldots, s_{K-1}, s_0, \ldots, s_{k-1}). \quad (11)$$

We then obtain the normalized throughput of station-$k$ (at steady state) to be given by

$$\rho_k = \frac{\bar{G}_k}{\bar{M} + \bar{R}}. \quad (12)$$

The mean (steady-state) packet delay at station-$k$, $\bar{D}_k$, can be computed from $\rho_k$, using the well-known expression for a single buffer station: $\bar{D}_k = 1/\rho_k - 1/\lambda_k$. This relationship is based on the observation that station-$k$ transmits a single packet every average period of duration $\lambda_k^{-1} + \bar{D}_k$, so that $\rho_k = 1/(\lambda_k^{-1} + \bar{D}_k)$. The mean packet waiting time at station-$k$, denoted as $\bar{W}_k$, is thus given by

$$\bar{W}_k = \frac{1}{\rho_k} - \frac{1}{\lambda_k} - 1. \quad (13)$$

4. Simplified analysis for a symmetric system

While the analysis presented in the previous section yields an exact solution to the limiting state distributions $\{\pi_k(s_k)\}$, this analytic method requires $3^K$ states to describe a system of $K$ stations. Since the number of
states grows exponentially with $K$, this exact analysis is applicable only for a limited number of stations, e.g., $K \leq 10$.

In this section, we present a simplified analysis for a symmetric system with parameters $\lambda_k = \lambda$ and $T_k = T$, $k = 0, 1, \ldots, K - 1$. By using the counting processes which represent the numbers of transmitted and deferred packets in a token rotation cycle, this simplified analysis can be used to approximately analyze a system with a large number of stations, e.g., $K = 20$.

### 4.1. The counting process

We introduce the following random (counting) variables:

- $X_k^n$: the overall number of packets transmitted at stations $0, 1, \ldots, k - 1$ during $[\tau^n_0, \tau^n_k)$,
- $Y_k^n$: the overall number of packets deferred at stations $0, 1, \ldots, k - 1$ during $[\tau^n_0, \tau^n_k)$,
- $Q_k^n$: the overall number of packets transmitted at stations $k, \ldots, K - 1$ during $[\tau^n_k, \tau^n_{k+1})$, $k = 0, 1, \ldots, K - 1$.

We note that $X_k^n = \sum_{i=0}^{k-1} I(s_i^n = 1)$, $Y_k^n = \sum_{i=0}^{k-1} I(s_i^n = 2)$, $Q_k^n = \sum_{i=k}^{K-1} I(s_i^n = 1)$, and $X_k^n + Q_k^n = M^n_k$. We examine this symmetric system during the token rotation cycles with respect to station-0. The state of the system during the $n$th token rotation cycle with respect to station-0, $[\tau^n_0, \tau^n_{n+1})$, $n \geq 0$, is described by a vector $(X_k^n, Y_k^n)$. The (counting) state process $(X_K, Y_K) = \{(X_k^n, Y_k^n) \mid n \geq 0\}$ constructs a discrete-time Markov chain over the state space $A'$:

$$A' = \{(x, y) \mid x = 0, 1, \ldots, [K, T], y = 0, 1, \ldots, K - x\}.$$  

The limiting distribution of the counting states $(X_k^n, Y_k^n)$, $n \geq 0$, is defined as

$$\hat{\pi}(x, y) = \lim_{n \to \infty} P(X_k^n = x, Y_k^n = y), \quad (x, y) \in A'. \quad (14)$$

The underlying Markov chain has a finite state space and is noted to be irreducible and aperiodic. Therefore a unique limiting (steady-state) distribution exists.

The steady-state probabilities satisfy the following set of balance equations:

$$\hat{\pi}(x, y) = \sum_{(i, j) \in A'} \hat{\pi}(i, j) \cdot p_k(x, y \mid i, j), \quad (x, y) \in A', \quad (15)$$

$$\sum_{(x, y) \in A'} \hat{\pi}(x, y) = 1, \quad (16)$$

where $\{(x, y) \mid (i, j)\}$ denote the associated transition probabilities, defined as follows:

$$p_k(x, y \mid i, j) = P(X_k^{n+1} = x, Y_k^{n+1} = y \mid X_k^n = i, Y_k^n = j), \quad (x, y), (i, j) \in A'. \quad (17)$$

We describe the evaluations of $\{p_k(x, y \mid i, j)\}$ in the following section.

### 4.2. The approximate transition probabilities

To exactly compute the transition probabilities $\{p_k(x, y \mid i, j)\}$, we need to go through a total of $3^K \times 3^K$ state transitions (i.e. the overall number of state transitions from $S_0^n$ to $S_{n+1}^n$). As the number of stations in the system, $K$, is large, numerical evaluations of $\{p_k(x, y \mid i, j)\}$ become computationally intractable. We develop here an efficient method to approximately evaluate $\{p_k(x, y \mid i, j)\}$ by using recursive calculations of the conditional probabilities $\{p_k^{(l)}(x, y \mid i, j, q)\}$, defined as follows:

$$p_k^{(l)}(x, y \mid i, j, q) = P(X_k^{n+1} = x, Y_k^{n+1} = y \mid X_k^n = i, Y_k^n = j, Q_k^n = q, R^n = r_0 + l), \quad (x, y, i, j, q) \in I_k, \quad l = 0, 1, \ldots, R_{\max}, \quad k = 1, \ldots, K, \quad (17)$$
where $R^n$ denotes the $n$th token walk time (slot) with respect to station-0 and

$$T_k = \left\{ \begin{array}{l}
  x = 0, 1, \ldots, [k, T], \ y = 0, 1, \ldots, k - x, \\
i = 0, 1, \ldots, [k, T], \ j = 0, 1, \ldots, k - i, \\
q = 0, 1, \ldots, [K - k, T - x, T - i]
\end{array} \right\}, \ k = 1, \ldots, K. \quad (18)
$$

Since $Q_k = 0$, we have from Eq. (17)

$$p_{K}^{(l)}(x, y \mid i, j, 0) = p(X_{k+1}^n = x, Y_{k+1}^n = y \mid X_k^n = i, Y_k^n = j, Q_k^n = 0, R^n = r_0 + l)
$$

$$= p(X_{k+1}^n = x, Y_{k+1}^n = y \mid X_k^n = i, Y_k^n = j, R^n = r_0 + l). \quad (19)$$

The transition probabilities, $\{p_K(x, y \mid i, j)\}$, can thus be computed by averaging $\{p_{K}^{(l)}(x, y \mid i, j, 0)\}$ over $l$ (which corresponds to a walk time of $r_0 + l$) as follows:

$$p_K(x, y \mid i, j) = \sum_{l=0}^{\text{max}} p_{K}^{(l)}(x, y \mid i, j, 0) \cdot r(l), \ (x, y, (i, j) \in A'. \quad (20)$$

We present in the following, a recursive procedure which is used to approximately evaluate $\{p_{K}^{(l)}(x, y \mid i, j, 0)\}$ for each $l \in \{0, 1, \ldots, R_{\text{max}}\}$. To simplify the recursive expressions involved in computing $\{p_{K}^{(l)}(x, y \mid i, j, q)\}$ in Eq. (22), we assume uniform distributions of the $i$ transmitted packets and the $j$ deferred packets among the $k$ stations (see the Appendix for the derivation of Eq. (22)).

**Procedure 2.** Recursive computations of $\{p_{K}^{(l)}(x, y \mid i, j, 0)\}$, given $l \in \{0, 1, \ldots, R_{\text{max}}\}$.

**initialization** $k = 1,$

$$p_1^{(l)}(0, 0|0, 0, q) = h_{0,0}^{(l)}(q),$$
$$p_1^{(l)}(0, 1|0, 0, q) = h_{0,2}^{(l)}(q),$$
$$p_1^{(l)}(0, 0|0, 1, q) = 0,$$
$$p_1^{(l)}(0, 1|0, 1, q) = h_{2,2}^{(l)}(q), \quad q = 0, 1, \ldots, [K - 1, T]; \quad (21)$$

$$p_1^{(l)}(1, 0|0, 0, q) = h_{0,0}^{(l)}(q),$$
$$p_1^{(l)}(0, 0|1, 0, q) = h_{1,0}^{(l)}(q),$$
$$p_1^{(l)}(1, 0|1, 0, q) = h_{1,1}^{(l)}(q),$$
$$p_1^{(l)}(0, 1|1, 0, q) = h_{1,2}^{(l)}(q),$$
$$p_1^{(l)}(1, 0|0, 1, q) = h_{2,0}^{(l)}(q), \quad q = 0, 1, \ldots, [K, T] - 1.$$

**recursive computations** For $k = 2, \ldots, K,$
The transition probabilities \( \{ h_{i,j}^{(l)}(v) \} \) in Eq. (22) can be obtained by setting \( r(l) = 1, r(m) = 0 \) for \( m \neq l \), in the expressions of \( \{ \tilde{h}_{i,j}^{(l)}(v) \} \) in Eq. (6). We have

\[
\begin{pmatrix}
  f^{(0)}(v) & g^{(0)}(v) & 1 - f^{(0)}(v) - g^{(0)}(v) \\
  f^{(1)}(v) & g^{(1)}(v) & 1 - f^{(1)}(v) - g^{(1)}(v) \\
  0 & r(0)(v) & 1 - f(0)(v)
\end{pmatrix}
\]

\[ i,j \in \{0,1,2\}, \quad v \in \{0,1,\ldots,K-1\}, \quad l \in \{0,1,\ldots,R_{\text{max}}\}, \]

where the elements of the matrix \( \{ h_{i,j}^{(l)}(v) \} \), given \( l \in \{0,1,\ldots,R_{\text{max}}\} \), are noted to be (see Eqs. (7)-(9))

\[
f^{(l)}(v) = e^{-\lambda(v+\rho_v+l)}, \quad v = 0,1,\ldots,K-1.
\]

\[
g^{(l)}(v) = (1 - e^{-\lambda(v+\rho_v+l)}) I(v < T - l - i), \quad i = 0,1,\ldots,K-1,
\]

\[
r^{(l)}(v) = I(v < T - l), \quad v = 0,1,\ldots,K-1.
\]

We note the following. (1) In writing the right-hand side of Eq. (22), we assume \( p^{(l)}_{k-1}(x,y \mid i,j,q) = 0 \), \( (x,y,i,j,q) \notin T_{k-1} \). (2) Let \( |T_k| \) denote the number of elements in \( T_k, k = 1,\ldots,K \). For \( T \geq K \), the required computer memory space, denoted as \( |T_{K}^{\text{max}}| \), is given by

\[
|T_{K}^{\text{max}}| = \max_{k=1,\ldots,K} |T_k| - \max_{k=1,\ldots,K} \left\{ \frac{(k+1)(k+2)}{2}(K-k+1) \right\},
\]

which is an upper bound of the required computer memory space for \( T < K \). Since \( |T_{K}^{\text{max}}| \) is on the order of \( O(K^4) \) (instead of \( O(K^5) \)), the above recursive procedure can be used to efficiently compute \( \{ p^{(l)}_k(x,y \mid i,j,q) \} \) for a relatively large number of stations, e.g., \( K = 20 \).

4.3. Iterative calculations of \( \{ \hat{\pi}(x,y) \} \)

We present here an iterative procedure for the calculations of the limiting state distribution \( \{ \hat{\pi}(x,y) \} \), using the approximate transition probabilities \( \{ p^{(l)}_k(x,y \mid i,j) \} \) obtained from Eq. (20).

**Procedure 3.** Iterative computations of \( \{ \hat{\pi}(x,y) \} \).

- **(initialization)** \( i = 0 \),
  \( \hat{\pi}^{(0)}(0,0) \leftarrow 1 ; \hat{\pi}^{(0)}(x,y) \leftarrow 0 ; (x,y) \neq (0,0) ; (x,y) \in A' \).
• (iteration) $i \leftarrow i + 1,$
  calculate $\{\hat{p}^{(i)}(x, y)\}$, given $\{\hat{p}^{(i-1)}(x, y)\}$, according to Eq. (15).
• (convergence check) if $(\sum_{(x, y) \in A} |\hat{p}^{(i)}(x, y) - \hat{p}^{(i-1)}(x, y)| > \varepsilon)$ repeat (iteration), else stop; we obtain the limiting state distribution $\{\hat{p}^{(i)}(x, y)\}$.

4.4. Performance statistics

Let $X(i), i = 0, 1, \ldots, [K, T], \ldots, [K, T]$ denote the probability that $i$ packets are transmitted during a token rotation cycle. $\{X(i)\}$ can be obtained from $\{\hat{p}(i, j)\}$ to be given by

$$X(i) = \sum_{j=0}^{K-i} \hat{p}(i, j), \quad i = 0, 1, \ldots, [K, T]. \quad (27)$$

We use $Y(j), j = 0, 1, \ldots, K$, to denote the probability that $j$ packets are deferred during a token rotation cycle. $\{Y(j)\}$ can be computed as follows:

$$Y(j) = \sum_{i=0}^{[K-jT]} \hat{p}(i, j), \quad j = 0, 1, \ldots, K. \quad (28)$$

Let $\bar{X} (\bar{Y})$ denote the average number of transmitted (deferred) packets during a token rotation cycle. The normalized throughput of the symmetric $K$-station system is thus given by

$$\rho = \frac{\bar{X}}{\bar{X} + \bar{Y}}. \quad (29)$$

Similar to Eq. (13), the mean packet waiting time can be computed as follows:

$$\bar{W} = \frac{K}{\rho} - \frac{1}{\lambda} - 1, \quad (30)$$

noting each station's throughput level to be equal to $\rho/K$.

5. Applications

We illustrate the application of the analytic approach developed above by employing the following numerical examples. In the analytic approach and simulations, the equivalent timing threshold at each station is set equal to an exact multiple of the slot time.

5.1. A symmetric system

First, we consider a symmetric token ring system with 20 stations ($K = 20$). The packet transmission time (slot) is set equal to 0.36 msec (corresponding to the transmission of a packet containing 4.5 Kbytes). The walk time of the system is selected to be 1 (slot) (corresponding to a geographic span of 72 km; $r_0 = 1$). Each station in the system is provided with an asynchronous priority-1 class of service ($T_{pri-1} = TTRT$). The equivalent timing threshold $T = TTRT - r_0$ indicates the maximum value to be assumed by the number of transmitted packets (which is equal to the number of stations that are allowed to transmit, since each station has a single packet buffer) during a token rotation cycle. We apply the simplified (counting process) analysis presented in Section 4 to investigate the queueing behavior of this symmetric system.
In Fig. 1, we examine the mean packet waiting time under various network throughput levels. The equivalent timing threshold \( T \) is varied from \( T = 6 \) (which corresponds to the value of \( TTTR = 7 \) slots = 2.52 msec) to \( T > K = 20 \) (which corresponds to the exhaustive service). By increasing the value of \( T \), we increase the number of stations that are allowed to transmit in a token rotation cycle. Consequently, the mean packet waiting time is decreased and the maximum network throughput level is increased as we vary the value of \( T \) from \( T = 6 \) to \( T > 20 \). We note that our simplified analysis yields performance results which are close to those obtained from simulations.

In Figs. 2-4, we set the equivalent timing threshold \( f \) to be equal to 6 (slots) to investigate the queueing performance of this system.

In Fig. 2, we show the convergence rate (in terms of \( N \), the number of iterations required to achieve a satisfactory level of convergence) of Procedure 3 with respect to the network throughput \( \rho \), under various packet arrival rates \( \lambda \). The convergence level of Procedure 3 is set to be \( \varepsilon = 10^{-6} \). We observe that the required number of iterations \( N \) reaches its peak value as the network starts to saturate (\( \rho = 0.8 \) and \( N = 45 \) for \( T = 6 \); \( \rho = 0.9 \) and \( N = 32 \) for the exhaustive service, \( T > 20 \)) and is consistently small over the entire range of system offered load.

In Fig. 3, we illustrate the distributions of the number of packets transmitted in a token rotation cycle (\( \{X(i)\} \)) under various network throughput levels. We observe that the probability distributions \( \{X(i)\} \) vary approximately from exponential (\( \rho = 0.3 \), lightly loaded), uniform (\( \rho = 0.8 \), highly loaded), to reversed exponential (\( \rho \geq 0.83 \), heavily loaded) distributions. Our analytic results are shown to faithfully predict these probability distributions over various network loading conditions.

In Fig. 4, we illustrate the distributions of the number of packets deferred in a token rotation cycle (\( \{Y(i)\} \)) under various network throughput levels. When the system is lightly loaded (\( \rho \leq 0.47 \), packets arriving at the stations are served almost exhaustively. As the system throughput level is increased (\( \rho = 0.63, 0.74 \)), the percentage of the number of packets deferred during a token rotation cycle is increased. When the network saturates (\( \rho > 0.8 \)), this number of deferred packets increases significantly. We observe that our simpli-
Fig. 3. Probability distributions of the number of packets transmitted in a token rotation cycle \( \{X(i)\} \) under various system throughput levels.

Fig. 4. Probability distributions of the number of packets deferred in a token rotation cycle \( \{Y(i)\} \) under various system throughput levels.

Fig. 5. The average number of packets deferred in a token rotation cycle \( \bar{Y} \) vs. system throughput \( \rho \) under various timing thresholds \( T \).

ified analysis employing the counting process, exhibits performance results almost identical to those obtained from simulations.

In Fig. 5, we compare the average number of packets deferred in a token rotation cycle \( \bar{Y} \) under various network throughput levels. The equivalent timing threshold \( T \) is varied from \( T > 20 \) (exhaustive service), to \( T = 12 \) and then to \( T = 6 \). When the system is under exhaustive service \( (T > 20; \rho < 0.7, T = 12; \)
5.2. A system with two priority classes of service

We next consider a token ring system with 10 stations. The stations around the ring are alternately provided with asynchronous priority-1 and priority-2 services. The walk time and the packet transmission time are the same as those given in the previous example. The stations are assumed to be symmetrically loaded. The asynchronous priority thresholds are set to be: $T_{p1} = 3.24$ (msec) and $T_{p2} = 2.16$ (msec). The equivalent timing thresholds ($T$) for priority-1 and priority-2 stations, respectively, are thus given by 8 (slots) and 5 (slots). We use the exact analysis presented in Section 3 to study the queueing performance of this system with two priority classes of stations.

In Fig. 6, we investigate the mean packet waiting time under various network throughput levels. When the network throughput is less than 0.5, priority-1 and priority-2 packets are served almost exhaustively. The performance curve for the mean waiting time of priority-1 packets coincides with that of priority-2 packets. As the offered load to the network increases, the effects of the asynchronous timing thresholds $T_{p1}$ and $T_{p2}$ become prominent. By limiting packet transmissions at the low priority (priority-2) stations, the mean waiting time of a priority-1 packet increases slightly while priority-2 packets experience much longer delays.
Fig. 9. Mean waiting time ($\bar{W}$) vs. system throughput ($\rho$) for the FDDI system with a host computer and a cluster of terminals, $TTRT=8$ msec.

Fig. 10. Throughput levels of the host computer ($\rho_h$), terminals ($\rho_t$), and the system ($\rho_c + \rho_t$) vs. packet arrival rate at the host computer ($\lambda_c$).

In Fig. 7, we show the throughput performance of the system under various (station) packet arrival rates ($\lambda$). As we increase $\lambda$ from 0 to 0.08, the throughput levels of both classes of stations are increased. However, when we further increase $\lambda$ above 0.1, the throughput level of priority-1 traffic increases while the throughput level of priority-2 traffic is decreased. As the network throughput saturates, both the overall system throughput and the throughput of priority-1 traffic reach their maximum values, while the throughput of priority-2 traffic diminishes.

In Fig. 8, we examine the average number of packets deferred in a token rotation cycle ($Y$) under various network throughput levels. Priority-1 packets are served exhaustively ($Y = 0$ for $T = 8$) under the system set-up. When the system is lightly loaded ($\rho < 0.5$), priority-2 packets are served exhaustively. As we increase the system throughput, the average number of deferred priority-2 packets in a token rotation cycle increases, so that the service for priority-2 packets further deviates from exhaustive service.

5.3. A system with a host computer and a cluster of terminals

We consider an FDDI network in which a host computer is connected to a cluster of ten terminals. The packet transmission time (slot) is selected to be equal to 0.36 msec. The walk time of the system is equal to 1 (slot) ($r_0 = 1$). The buffer sizes of the host computer and each terminal are set equal to 20 and 1 (packet), respectively. Packets arrive at each terminal according to a Poisson process with rate $\lambda_t = 0.03$. The terminals are assumed to be symmetrically loaded. Packets arrive at the host computer according to a Poisson process with rate $\lambda_h$. We vary the traffic load contributed by the host computer from 0.03 to 0.9 ($\lambda_c = 0.03 \sim 0.9$). The host computer and the terminals are served under asynchronous priority-1 service with a timing threshold of $T_{pr1} = TTRT = 8$ msec = 22 slots.

We use the simplified (counting process) method presented in Section 4 to analyze the queueing behavior of the terminals. For the analysis of the host computer, we employ a model which represents the station as an $M/G/1/N$ queueing system with vacations and dynamic time-limited service, as presented in [13]. Combining the above-mentioned analyses for the terminals and the host computer, we develop the following iterative
procedure for the analysis of the complete system. The walk time distribution \( \{ r(i) \} \) is given by the token dwell time distribution at the host computer, using the \( M/G/1/N \) vacation model. The distribution of the number of packets transmitted at the terminals in a token rotation cycle \( \{ X(i) \} \) is used as the token vacation time distribution in the \( M/G/1/N \) vacation modeling of the host computer. We compute alternatively the walk time distribution \( \{ r(i) \} \) and the distribution of the number of packets transmitted at the terminals in a token rotation cycle \( \{ X(i) \} \) until the variation of \( \{ r(i) \} \) between successive iterations is within a satisfactory level of convergence.

In Fig. 9, we investigate the delay-throughput performance at the stations connected to the FDDI network. When the network throughput \( \rho \) is less than 0.7, the token rotates around the ring without being timed out most of the time. Thus the system behaves like an exhaustive system, in which a station with the largest offered load has the least packet delay. Since \( \lambda_c > \lambda_t \), packets queued at the host computer experience lower mean waiting times than those at the terminals. As we increase the offered load at the host computer, the effect of the timing threshold \( TTR_T \) becomes prominent in limiting packet transmissions at the host computer. When the network is highly loaded \( (\rho > 0.8) \), we make the following observations. While packets queued at the (heavily loaded) host computer experience sharply increasing delays as the network saturates, packets queued at the (lightly loaded) terminals experience only slightly increased delays. This phenomenon stems from the dynamic bandwidth sharing process under which the stations are provided with fair access (i.e., equal sharing based) to the system bandwidth. By limiting transmissions at the heavily loaded stations during each token rotation cycle, the FDDI network guarantees a satisfactory level of service at the lightly loaded stations.

In Fig. 10, we show the throughput performance of the system. As we increase the packet arrival rate at the host computer \( (\lambda_c) \), the throughput at the terminals \( (\rho_t) \) is slightly decreased (caused by the single packet buffer capacity limitation and by bandwidth captured by the host computer) and the throughput levels of the host computer \( (\rho_c) \) and the system are increased. The maximum throughput levels of the host computer and the system are noted to be equal to 0.65 and 0.92, respectively.

In Fig. 11, we illustrate the walk time distribution \( \{ r(i) \} \), obtained from the \( M/G/1/N \) vacation model and...
from simulations.

In Fig. 12, we show the distribution of the number of packets transmitted at the terminals in a token rotation cycle \( \{X(i)\} \), obtained from the simplified (counting process) analysis as well as simulations. We observe that our analytic results are in general agreement with simulation results.

Appendix. Recursive expressions for calculating \( \{p_k^{(l)}(x, y \mid i, j, q)\} \)

We derive the recursive equations expressed by Eq. (22) for \( \{p_k^{(l)}(x, y \mid i, j, q)\} \), \( k = 2, \ldots, K - 1 \), in terms of \( \{p_k^{(l)}(x, y \mid i, j, q)\} \), given \( l \in \{0, 1, \ldots, R_{\text{max}}\} \). Since the index \( l \) is unchanged in the following derivations (for a given value of \( l \)), we simplify the expressions below by eliminating the notations involving \( l \):

1. We employ \( \{p_k(x, y \mid i, j, q)\} \) and \( \{h_{s,t}(v)\} \), respectively, to represent \( \{p_k^{(l)}(x, y \mid i, j, q)\} \) and \( \{h_{s,t}(v)\} \).

2. The conditional probabilities \( \{P(A^{n+1} \mid B^n)\} \) \( n \geq 0 \), associated with the transitions from the event \( B^n \) to the event \( A^{n+1} \) in successive \( (n\text{th and } (n + 1)\text{st}) \) token rotation cycles, are used to represent \( \{P(A^{n+1} \mid B^n, R^n = r_0 + l)\} \).

By definition, we have

\[
p_k(x, y \mid i, j, q) = P(X_k^{n+1} = x, Y_k^{n+1} = y \mid X_k^n = i, Y_k^n = j, Q_k^n = q) \\
= \sum_{s=0,1,2} P(s_{k-1}^n = s \mid X_k^n = i, Y_k^n = j, Q_k^n = q) \cdot P(X_k^{n+1} = x, Y_k^{n+1} = y \mid X_{k-1}^n = i - I(s = 1), Y_{k-1}^n = j - I(s = 2), s_{k-1}^n = s, Q_k^n = q) \quad (A.2)
\]

By assuming uniform distributions of the \( i \) transmitted packets and the \( j \) deferred packets among the \( k \) stations, we can evaluate approximately the first term of the right-hand side in Eq. (A.2) as follows. Note that \( s_{k-1}^n \) is independent of \( Q_k^n \) \( (= \sum_{t=1}^{k-1} I(s = 1)) \), i.e., the state of a station does not depend on the future evolutions of the states at the other stations.

\[
P(s_{k-1}^n = 1 \mid X_k^n = i, Y_k^n = j, Q_k^n = q) = P(s_{k-1}^n = 1 \mid X_k^n = i, Y_k^n = j) \\
= \frac{(k-1)!/(i-1)!/(k-i-j)!}{k!/i!(k-i-j)!} = \frac{i}{k}. \quad (A.3)
\]

Similarly, we have

\[
P(s_{k-1}^n = 0 \mid X_k^n = i, Y_k^n = j, Q_k^n = q) = 1 - \frac{i+j}{k}, \quad (A.4)
\]

\[
P(s_{k-1}^n = 2 \mid X_k^n = i, Y_k^n = j, Q_k^n = q) = \frac{j}{k}. \quad (A.5)
\]

The second term of the right-hand side in Eq. (A.2) can be expanded as follows:
\[ P \left( X_k^{r+1} = x, Y_k^{r+1} = y \mid X_{k-1}^r = i - l(s = 1), Y_{k-1}^r = j - l(s = 2), s_{k-1}^r = s, Q_k^r = q \right) \]

\[ = \sum_{s' = 0, 1, 2} P \left( X_{k-1}^{r+1} = x - l(s' = 1), Y_{k-1}^{r+1} = y - l(s' = 2), s_{k-1}^{r+1} = s' \right) \]

\[ - P \left( s_{k-1}^{r+1} = s' \mid X_{k-1}^r = i - l(s = 1), Y_{k-1}^r = j - l(s = 2), s_{k-1}^r = s, Q_k^r = q \right) \]

Since \( s_{k-1}^{r+1} \) depends only on \( s_{k-1}^r \) and \( Q_k^r + X_k^{r+1} \), we have

\[ P \left( s_{k-1}^{r+1} = s' \mid X_{k-1}^r = i - l(s = 1), Y_{k-1}^r = j - l(s = 2), s_{k-1}^r = s, Q_k^r = q \right) \]

\[ = h_{s,s'}(q + x - l(s' = 1)). \]  

(A.6)

Noticing that \( Q_k^r = Q_k^r + I(s_{k-1}^r = 1) \), we obtain

\[ P \left( X_{k-1}^{r+1} = x - l(s' = 1), Y_{k-1}^{r+1} = y - l(s' = 2) \right) \]

\[ = P \left( X_{k-1}^r = i - l(s = 1), Y_{k-1}^r = j - l(s = 2), s_{k-1}^r = s', s_{k-1}^r = s, Q_k^r = q \right) \]

\[ = p_{k-1}(x - I(s' = 1), x - I(s' = 2) \mid i - I(s = 1), j - I(s = 2), q + I(s = 1)). \]  

(A.7)

Combining Eqs. (A.2)–(A.8), we conclude the recursive computational procedure for \( \{p_k(x, y, i, j, q)\} \) as expressed in Eq. (22).

References

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