Delay analysis for forward signaling channels in wireless cellular network

Izhak Rubin and Cheon Won Choi

Department of Electrical Engineering, University of California at Los Angeles, Los Angeles, CA 90095-1594, USA

We consider connection-oriented wireless cellular networks. Such second generation systems are circuit-switched digital networks which employ dedicated radio channels for the transmission of signaling information. A forward signaling channel is a common signaling channel assigned to carry the multiplexed stream of paging and channel-allocation packets from a base station to the mobile stations. Similarly, for ATM wireless networks, paging and virtual-circuit-allocation packets are multiplexed across the forward signaling channels as part of the virtual-circuit set-up phase. The delay levels experienced by paging and channel-allocation packets are critical factors in determining the efficient utilization of the limited radio channel capacity. A multiplexing scheme operating in a “slotted mode” can lead to reduced power consumption at the handsets, but may in turn induce an increase in packet delays. In this paper, focusing on forward signaling channels, we present schemes for multiplexing paging and channel-allocation packets across these channels, based on channelization plans, access priority assignments and paging group arrangements. For such multiplexing schemes, we develop analytical methods for the calculation of the delay characteristics exhibited by paging and channel-allocation packets. The resulting models and formulas provide for the design and analysis of forward signaling channels for wireless network systems.

1. Introduction

We consider connection-oriented wireless cellular networks. Such second generation systems, e.g., IS-54 (Electronic Industry Association Interim Standard 54), IS-95 and GSM (European Global System for Mobile Communications), are circuit-switched digital networks which employ dedicated radio channels for the transmission of signaling information [2–6,11,12]. A forward signaling channel is a common signaling channel assigned to carry the multiplexed stream of paging and channel-allocation packets from a base station to mobile stations. Similarly, for ATM wireless networks, paging and virtual-circuit-allocation packets are multiplexed across the forward signaling channels as part of the virtual-circuit set-up phase. Paging packets are broadcasted by the base station across the forward signaling channel to alert a mobile station to an incoming call. Upon receipt of a response from the destined mobile, or upon receipt of a connection request from a call initiating mobile, the base station selects traffic channels to be allocated to the connection (when admitted), and transmits a channel-allocation packet across the forward signaling channel. In designing a multiplexing scheme for the forward signaling channel, it is necessary to investigate the following factors.

1. The queueing and transmission delays incurred by channel-allocation packets must be properly limited. For a call initiated by a mobile, the channel-request packet is many times transmitted across a reverse signaling channel which employs a contention oriented medium access control algorithm (such as an ALOHA protocol). In this case, the channel-allocation packet also serves as a positive acknowledgement message. If the latter is not received in time, the channel-request packet will have to be retransmitted, leading to degradation in the throughput efficiency of the signalling channels.

2. For calls destined to a mobile, the base station broadcasts paging packet(s) to alert the destination mobile. To avoid timeout of the initiating party, and to yield acceptable circuit set-up times, it is necessary to also limit the delays incurred by paging packets.

3. To reduce power consumption at the mobile’s handset, a “slotted mode” operation is available. Under such an operation, an inactive mobile enters a “sleep” state. If it will then listen to prescribed forward signaling channel(s) only during certain assigned time slots. Hence, the base station will transmit paging packets destined to Group-k users only during assigned Group-k slots in each time frame. In turn, channel-allocation packets can be transmitted during any time slot.

The multiplexing mechanism must be designed to properly integrate these features. In this paper, we investigate the delay performance exhibited by such packets in conjunction with the power consumption level at the handset, under various multiplexing algorithms. Our results provide an important basis for the evaluation and design of the signaling subsystems.

In section 2, we describe the candidate multiplexing schemes. Each of these schemes is characterized by a distinctive channelization plan, access priority assignment and paging group arrangement. In sections 3 and 4, we develop analytical methods for the calculation of delay distributions for paging and channel-allocation packets, under the multiplexing schemes described in section 2. Section 5 is devoted to numerical examples and performance comparisons. These studies provide for the optimization of the
packet delay performance incorporating the relative power consumption level required at the handset.

2. Multiplexing schemes for the forward signaling channel

The forward signaling channel is used for the transport of CA (channel-allocation) and PG (paging) packets from the BS (base station) to the MS’s (mobile stations), e.g., users, handsets. CA and PG packets are multiplexed and are transmitted across this forward signaling channel established over the underlying radio channel. We assume time to be divided into frames and a frame to be subdivided into slots. We assume that CA and PG packets have the same fixed length and that it takes a single slot time to transmit a packet. (A packet transmission starts at the beginning of a slot.) It is possible to multiplex CA packets and PG packets across the forward signaling channel in accordance with many different schemes. The following factors have been considered by many implementations [5]:

1. Use of access priorities.
2. Paging groups.
3. Assignment of slots for the exclusive transmission of CA packets.

High access priority can be assigned to either CA or PG packets. A nonpreemptive service discipline is then used, so that the transmission of low priority packet is not interrupted at the arrival of a high priority packet.

For reducing power consumption at MS’s, the MS’s are associated with a number of PG groups. For example, MS’s can be arranged into separate groups in accordance with their address identities. We assume in our models that each MS is a member of a single designated PG group. A single slot per frame is assigned to each PG group for the transmission of a PG packet belonging to that group. In this manner, a MS needs to listen to broadcasting paging information only during those slots dedicated to its PG group.

Priority assignments depend on the desired delay-throughput network performance characteristics. The system designer also prescribes target delay limits for PG and CA packets. To reduce incurred delay levels for CA packets, a number of slots per frame may be assigned for the exclusive transmission of such packets. (These slots can be uniformly distributed across the frame to reduce average frame latencies.) Such a reduction in the CA packet delay is achieved at the expense of PG packets whose delay values increase.

The various multiplexing schemes stated in this paper are summarized in Table 1. The following triplet designation is used: under scheme p.m.n, p = CA (p = PG) indicates high access priority is given to CA (PG) packets. The second digit m represents the number of PG groups (m_{PG}) and the third one (n) is defined as the number of slots per frame allocated for the exclusive transmission of CA packets (m_{CA}).

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Priority assignment</th>
<th>High priority for CA packets</th>
<th>High priority for PG packets</th>
<th>High priority for PG packets</th>
</tr>
</thead>
<tbody>
<tr>
<td>CA,m_{PG},0</td>
<td>The number of PG groups</td>
<td>m_{PG} ≥ 1</td>
<td>m_{PG} ≥ 1</td>
<td>m_{PG} ≥ 1</td>
</tr>
<tr>
<td></td>
<td>The number of slots exclusively assigned for CA packets</td>
<td>m_{CA} = 0</td>
<td>m_{CA} = 0</td>
<td>m_{CA} ≥ 0</td>
</tr>
</tbody>
</table>

In sections 3 and 4, we analyze the delay times of PG and CA packets under class CA and class PG, respectively.

3. Packet delay analysis for class CA: high priority for channel-allocation packets

The region covered by a wireless cellular network is partitioned into cells, and a BS is located in each cell. The BS in a cell delivers CA (channel-allocation) and PG (paging) packets to MS’s residing in the cell. These MS’s are divided into m_{PG} PG groups. Time is divided into frames and each frame is subdivided into slots. A frame consists of m_{PG} slots. In this section, we consider scheme CA,m_{PG},0, under which CA packets are granted higher priority. A single slot per frame is dedicated to a PG group, so that a frame consists of m_{PG} slots, i.e., m_F = m_{PG}. A CA packet can be transmitted in any slot. During a frame, a slot is used for the transmission of a PG packet destined to the corresponding PG group, if there is no CA packet waiting in the queue. At the BS, the arrival streams of CA packets and of PG packets for each group are modeled as independent Poisson processes. Let \lambda_{CA} denote the arrival rate of CA packets and \lambda_{PG} be the arrival rate of PG packets destined to Group-k. (The packet arrival rate is measured as the average number of packet arrivals per slot time.) Let \mathcal{X}^{\text{CA}}_{t} = \{X_{t}^{\text{CA}}, t ⩾ 0\} denote the system-size process for the CA packet queue at the BS and \mathcal{X}^{\text{PG}}_{t} = \{X_{t}^{\text{PG}}, t ⩾ 0\} be the BS’s system-size process for the Group-k PG packet queue. Thus, \mathcal{X}^{\text{CA}}_{t} (X_{t}^{\text{PG}}) represents the number of CA (Group-k PG) packets (waiting or being transmitted) resident at the queue of the BS at time t. Define the completion time of a packet to be the sojourn time of the packet at the top of the queue, i.e., it represents the time elapsed from the instant a packet is placed at the head of the queue to the instant it is transmitted across the channel.

3.1. Channel-allocation packet delay analysis

Under scheme CA,m_{PG},0, CA packets are transmitted in accordance with the slotted FCFS single channel policy. Equivalently, one can view their service system to be a
degnerate TDMA scheme, where a frame consists of single slot. Packet delay performance under TDMA schemes was extensively studied in [7–10]. In this section, we concisely summarize the procedure for deriving the Laplace–Stieltjes transform of the steady-state distribution function of the CA packet delay. These results will be also used in the following sections.

Consider a CA packet. If the packet is not the first packet to be served during a busy period associated with the process \( X_{CA} \), its completion time \( C_{CA} \) is equal to its service time. The packet’s service time is equal to a single slot time, so that \( C_{CA} = 1 \) a.s. Otherwise, the packet waits at the top of the queue until the start of the slot following its slot of arrival. This packet experiences an exceptional completion time \( K_{CA} \) which is different from the nominal completion time \( C_{CA} \). Suppose that a busy period starts at time \( A_1 \) and that the previous busy period ends at time \( R_0 \) \( (R_0 < A_1) \). (Note that arrivals take place in accordance with a continuous-time Poisson process, so that \( A_1 \) can occur at any time and not necessarily at the start of a slot.) Then,

\[
K_{CA} = 1 + \min\{m: m \geq A_1, m = 1, 2, \ldots \} - A_1
\]

\[
= 1 + \min\{m: m \geq [A_1 - R_0], m = 1, 2, \ldots \} - [A_1 - R_0].
\]

Note that \([A_1 - R_0]\) is the length of an exponentially distributed idle period. Let \( F_{K_{CA}}(x) \) denote the distribution function for \( K_{CA} \) and \( G_{K_{CA}}(s) \) be the corresponding Laplace–Stieltjes transform. Then,

\[
F_{K_{CA}}(x) = 0 \cdot I_{\{x<1\}} + \frac{e^{-\lambda CA}}{1 - e^{-\lambda CA}} [e^{-\lambda CA(1-x)} - 1] I_{\{1 \leq x < 2\}} + 1 \cdot I_{\{x \geq 2\}},
\]

\[
G_{K_{CA}}(s) = \frac{e^{-\lambda CA}}{1 - e^{-\lambda CA}} \lambda CA - s \times [e^{2\lambda CA - s} - e^{\lambda CA(1-s)}],
\]

for \( \text{Re}(s) \geq 0 \). Since the completion times of CA packets are statistically independent, \( X_{CA} \) can be regarded as the system-size process of an M/G/1 queueing system in which the arrival rate is \( \lambda_{CA} \), and the packet service time is equal to \( K_{CA} \) for a packet which is the first one to be served in a busy period, while it is equal to \( C_{CA} \) for other packets. Using an embedded Markov chain analysis method, we can obtain the probabilistic properties of the CA packet delay time. Let \( Z = \{Z_n, n \geq 1\} \) be the embedded chain of \( \{X_{CA}^t, t \geq 0\} \) at departure points, where \( Z_n \) represents the CA packet system-size following the departure of the \( n \)th CA packet. This embedded process is a Markov chain satisfying the relation

\[
Z_{n+1} \overset{d}{=} N_{CA}(K_{CA}) I_{\{Z_n = 0\}} + [Z_n + N_{CA}(C_{CA}) - 1] I_{\{Z_n \geq 1\}},
\]

where \( N_{CA}(x) \) represents the number of CA packet arrivals in a time interval of length \( x \). Set \( \rho_{CA} = \lambda_{CA}E(C_{CA}) \) to denote the traffic intensity of the CA packet queue at the BS. We assume \( \rho_{CA} < 1 \), so that \( Z \) is an irreducible and positive-recurrent Markov chain. Hence, \( Z \) has a steady-state distribution denoted as \( \{f_Z(i), i \geq 0\} \). Let \( g_Z(z) \) denote its generating function. We have [10]

\[
g_Z(z) = \frac{f_Z(0)z G_{K_{CA}}(\lambda_{CA}(1 - z)) - G_{C_{CA}}(\lambda_{CA}(1 - z))}{z - G_{C_{CA}}(\lambda_{CA}(1 - z))}, \tag{3}
\]

where \( G_{K_{CA}}(s) \) is given by (2) and \( G_{C_{CA}}(s) = E(e^{-sC_{CA}}) = e^{-s} \). By normalization, \( g_Z(1) = 1 \), yielding:

\[
f_Z(0) = \frac{1 - \lambda_{CA}E(C_{CA})}{1 - \lambda_{CA}[E(C_{CA}) - E(K_{CA})]]. \tag{4}
\]

Let \( \{D_n, n \geq 1\} \) be the sequence of CA packet delay times. By invoking Lindley’s theorem for GI/G/1 queueing system [1], if \( \rho_{CA} < 1 \), then there exists a steady-state random variable \( D_{CA} \) such that \( D_n \overset{d}{\rightarrow} D_{CA} \) as \( n \to \infty \). Let \( F_{D_{CA}}(x) \) denote the distribution function of \( D_{CA} \) and \( G_{D_{CA}}(s) \) be its Laplace–Stieltjes transform. Since \( Z_n = N_{CA}(D_n) \), we conclude from (3)–(4):

\[
G_{D_{CA}}(s) = \frac{1 - \lambda_{CA}E(C_{CA})}{1 - \lambda_{CA}[E(C_{CA}) - E(K_{CA})]} \times \frac{[\lambda_{CA} - s]G_{K_{CA}}(s) - \lambda_{CA}G_{C_{CA}}(s)}{\lambda_{CA}[1 - G_{C_{CA}}(s)] - s}. \tag{5}
\]

From eq. (5), we can calculate the moments of delay time of a CA packet at steady-state. For example, the mean CA packet delay is given by

\[
E(D_{CA}) = \frac{2E(K_{CA}) - \lambda_{CA}[E(C_{CA})^2] - E((K_{CA})^2)]}{2[1 - \lambda_{CA}[E(C_{CA}) - E(K_{CA})]]} \tag{6}
\]

\[
= \frac{\lambda_{CA}E(C_{CA})^2}{2[1 - \lambda_{CA}E(C_{CA})]} \tag{7}
\]

\[
= \frac{3}{2} \frac{\lambda_{CA}}{2(1 - \lambda_{CA})},
\]

which coincides with the result given in [8].

3.2. Paging packet delay analysis

First, consider a PG packet destined to Group-k which is not the first packet to be served during a busy period associated with the process \( X_{PG}^k \). Let \( C_{PG}^k \) denote the completion time of this Group-k PG packet. Then, \( C_{PG}^k \in \{mF, 2mF, \ldots\} \). For the purpose of computing the distribution for \( C_{PG}^k \), we introduce a sequence of random variables \( Y^k = \{Y_n^k, n \geq 0\} \) such that \( Y_n^k = X_{C_{PG}^k}^{n-m+k-1} \) for \( n \geq 0 \). Note that for a given \( k \in \{1, mF, \ldots\} \), \( Y^k \) represents the CA packet system-size at the start of the \( k \)th slot of each frame. We observe that \( Y^k \) is a homogeneous Markov chain on the state space \( S = \{0, 1, \ldots\} \) whose transition probabilities \( \{p(i, j), i, j \in S\} \) are given by

\[
p(0, j) = e^{-\lambda_{CA}j} \frac{\lambda_{CA}^j}{j!} \quad \text{for } j \in S,
\]

\[
p(i, j) = e^{-\lambda_{CA}j-i+1} \frac{\lambda_{CA}^j}{(j-i+1)!} \quad \text{for } j \geq i - 1, \ i \geq 1.
\]
Note that if $\lambda_{CA} < 1$, then $Y^k$ is an irreducible and positive-recurrent Markov chain. Hereafter, we assume $\lambda_{CA} < 1$. Let $(p^{(n)}(i,j), i,j \in S)$ denote the $n$-step transition probabilities for $Y^k$, $1 < k < m_F$. Set $\phi(n) = p^{(nm_F)}(0,0)$ for $n > 1$. For the underlying packet delay analysis, we require to calculate $\{\phi(n), n \geq 1\}$. We obtain

$$\phi(n) = e^{-\lambda_{CA} nm_F} \times \left[ 1 + \sum_{m=1}^{nm_F-1} \left( \frac{[nm_F]^m}{m!} - \frac{[nm_F]^{m-1}}{(m-1)!} \right) \lambda_{CA}^m \right],$$

for $n \geq 1$. The function $\phi(n)$ is derived in appendix. Set $f_{C_{\text{PG}}}(nm_F) = P(C_{\text{PG}} = nm_F)$ for $n \geq 1$. Then, for $n \geq 1$,

$$f_{C_{\text{PG}}}(nm_F) = \phi(n) - \sum_{m=1}^{n-1} \phi(m)f_{C_{\text{PG}}}(nm_F - m), \quad (6)$$

for $n \geq 2$.

Secondly, consider a Group-$k$ paging packet which is the first packet to be served during a busy period. Let $K_{\text{PG}}$ denote the completion time of this PG packet. Suppose that this PG packet arrives at time $A_1$ and departs at time $R_1$, and that the previous PG packet departs at time $R_0$ ($R_0 < A_1 < R_1$). Define a random variable $I^k = \min\{m: nm_F - 1 \geq |A_1 - R_0|, m \geq 1\}$. Note that $|A_1 - R_0|$ is the length of an exponentially distributed idle period. Set $f_{I^k}(m) = P(I^k = m)$ for $m \geq 1$. Then, we have

$$f_{I^k}(m) = \left[ 1 - e^{\lambda_C m_F} \right] I_{m=1} + \left[ e^{\lambda_C (m-1)m_F} - e^{\lambda_C m_F} \right] I_{m \geq 2} \quad (7)$$

By introducing the two random variables, $U^k = I^k m_F - A_1 + R_0$ and $V^k = R_1 - I^k m_F + R_0$, we partition the completion time $K_{\text{PG}}$ into two parts such that $K_{\text{PG}} = R_1 - A_1 = U^k + V^k$ (figure 1). Note that $U^k$ represents the frame latency delay component (relative to Group-$k$ frame starts) plus 1 slot, while $V^k$ is the residual delay time. The distribution function for $U^k$, $F_{U^k}(x)$ is given by

$$F_{U^k}(x) = 0 \cdot I_{x < 1} + \frac{e^{-\lambda_C m_F}}{1 - e^{-\lambda_C m_F}} \left[ e^{\lambda_C x} - e^{\lambda_C} \right] I_{1 \leq x < m_F} + \frac{e^{-2\lambda_C m_F}}{1 - e^{-\lambda_C m_F}} \left[ e^{\lambda_C x} - e^{\lambda_C} \right] I_{m_F \leq x < m_F + 1} + 1 \cdot I_{x \geq m_F + 1}. \quad (8)$$

The random variable $V^k$ assumes values which are non-negative integer multiples of $m_F$. Given $I^k$, the conditional probabilities $\{P(V^k = nm_F | I^k), n \geq 0\}$ satisfy the following recursive equation:

$$P(V^k = 0 | I^k) = \phi(I^k),$$

$$P(V^k = 1 | I^k) = \phi(I^k + 1),$$

and for $n \geq 2$,

$$P(V^k = nm_F | I^k) = \phi(I^k + n) - \sum_{m=1}^{n-1} \phi(m)P(V^k = [n-m]m_F | I^k).$$

Let $f_{V^k}(m)$ denote the mass function for $V^k$. Then,

$$f_{V^k}(nm_F) = \sum_{m=1}^{\infty} P(V^k = nm_F | I^k = i) f_{I^k}(m), \quad (9)$$

for $n \geq 0$, where $f_{I^k}(m)$ is given by (7). Let $F_{K^k_{\text{PG}}}(x)$ denote the distribution function for $K^k_{\text{PG}}$. From eqs. (8) and (9), we perform a convolution operation to yield

$$F_{K^k_{\text{PG}}}(x) = \sum_{n=0}^{\infty} F_{U^k}(x - nm_F) f_{V^k}(nm_F), \quad (10)$$

for $x \geq 1$. Eqs. (10) and (6) provide for the calculation of the distribution of the PG packet completion time.

Since the completion times of Group-$k$ PG packets are statistically independent over $k \in \{1, \ldots, m_{PG}\}$, we can adopt the same method as the one used in section 3.1 to obtain the probability distribution of the PG packet delay time. We assume that $\rho_{\text{PG}} = \lambda_{\text{PG}} E(C_{\text{PG}}) < 1$. Let $\{D^k_n, n \geq 1\}$ be the sequence of Group-$k$ PG packet delay times. Then, there exists a steady-state random variable $D^k_{\text{PG}}$ such that $D^k_n \xrightarrow{d} D^k_{\text{PG}}$ as $n \to \infty$. Let $G_{K^k_{\text{PG}}}(s)$ and $G_{C^k_{\text{PG}}}(s)$ denote Laplace–Stieltjes transforms of the distribution functions for $K^k_{\text{PG}}$ and $C^k_{\text{PG}}$, respectively. Let $F_{D^k_{\text{PG}}}(x)$ denote the distribution function for $D^k_{\text{PG}}$, and $G_{U^k_{\text{PG}}}(s)$ be the corresponding Laplace–Stieltjes transform. We obtain

$$G_{D^k_{\text{PG}}}(s) = \frac{1 - \lambda_{\text{PG}} E(C_{\text{PG}})}{1 - \lambda_{\text{PG}} [E(C_{\text{PG}}) - E(K_{\text{PG}})]} \times \frac{\lambda_{\text{PG}}}{s} [G_{K^k_{\text{PG}}}(s) - \lambda_{\text{PG}} G_{C^k_{\text{PG}}}(s)] \lambda_{\text{PG}}[1 - G_{C^k_{\text{PG}}}(s)] - s. \quad (11)$$
By differentiating eq. (11) and using the moments of the PG packet completion time which are obtained from eqs. (10) and (6), we can calculate the moments of the steady-state delay time of the Group-\(k\) PG packet.

4. Packet delay analysis for class PG: high priority for paging packets

Under scheme PG,\(m_{\text{PG}},m_{\text{CA}}\), higher priority is given to PG packets. The MS’s residing in a cell are divided into \(m_{\text{PG}}\) PG groups. A frame of the forward signaling channel consists of \(m_{\text{PG}}\) slots (used for the transmission of PG packets) and an additional set of \(m_{\text{CA}}\) slots. These \(m_{\text{CA}}\) slots are assigned for the exclusive transmission of CA packets.

Set \(m_{F} = m_{\text{PG}} + m_{\text{CA}}\). A PG packet must be transmitted across the frame’s slot which is allocated to the group in which its destination MS resides. If no PG packet is waiting in a PG packet queue, a CA packet can be transmitted using the slot allocated to this group. A CA packet can also be transmitted by using a slot belonging to the set of \(m_{\text{CA}}\) slots in each frame.

We adopt the same packet arrival model at the base station as defined in section 3. At the BS, the arrival streams of CA and PG packets for each group are modeled as independent Poisson processes with arrival rates \(\lambda_{\text{CA}}\) and \(\lambda_{\text{PG}}\) (\(1 \leq k \leq m_{\text{PG}}\)), respectively. Let \(X_{k}^{t} = \{X_{k}^{t}, t \geq 0\}\) denote the system-size process of PG packets destined to MS’s which belong to Group-\(k\) for \(1 \leq k \leq m_{\text{PG}}\), so that \(X_{k}^{t}\) represents the number of Group-\(k\) packets resident at the corresponding queue of the BS at time \(t\). Let \(X_{\text{CA}}^{t} = \{X_{\text{CA}}^{t}, t \geq 0\}\) be the BS’s system-size process of CA packets. The completion time of a packet is defined as the sojourn time of the packet at the top of the queue, i.e., it represents the time elapsed from the instant a packet is placed at the head of the queue to the instant it is transmitted across the channel.

4.1. Paging packet delay analysis

Under scheme PG,\(m_{\text{PG}},m_{\text{CA}}\), a PG packet destined to a MS which belongs to a PG group is transmitted by using a slot allocated to the group in a frame. Group-\(k\) PG packets are served in accordance with a FCFS policy for each \(1 \leq k \leq m_{\text{PG}}\). Let \(C_{\text{PG}}^{k}\) denote the completion time of a Group-\(k\) PG packet which is not the first packet to be served during a busy period associated with the process \(\lambda_{\text{PG}}^{k}\). Since a single slot per frame is assigned for each Group-\(k\) PG packet queue, we conclude that \(C_{\text{PG}}^{k} = m_{F}\) a.s., for all \(1 \leq k \leq m_{\text{PG}}\). Hence, the corresponding Laplace–Stieltjes transform, \(G_{C_{\text{PG}}^{k}}(s) = e^{-m_{F}s}\), for \(\text{Re}(s) \geq 0\). Let \(F_{K_{\text{PG}}^{k}}(x)\) denote the completion time of a Group-\(k\) PG packet which is the first packet to be served during a busy period. Let \(F_{K_{\text{PG}}^{k}}(x)\) denote the distribution function of \(K_{\text{PG}}^{k}\) and \(G_{K_{\text{PG}}^{k}}(s)\) be the corresponding Laplace–Stieltjes transform. Then, using the same approach employed for deriving eqs. (1) and (2) in section 3.1, we obtain

\[
F_{K_{\text{PG}}^{k}}(x) = 0 \cdot I_{\{x < 1\}} + \frac{e^{-\lambda_{\text{PG}}^{k}m_{F}}}{1 - e^{-\lambda_{\text{PG}}^{k}m_{F}}} \left[ e^{\lambda_{\text{PG}}^{k}x} - e^{\lambda_{\text{PG}}^{k}} \right] I_{\{1 \leq x < m_{F}\}} + \frac{e^{-2\lambda_{\text{PG}}^{k}m_{F}}}{1 - e^{-\lambda_{\text{PG}}^{k}m_{F}}} \left[ e^{\lambda_{\text{PG}}^{k}x} - e^{\lambda_{\text{PG}}^{k}} \right] I_{\{m_{F} \leq x < m_{F} + 1\}} + 1 \cdot I_{\{x \geq m_{F} + 1\}},
\]

for \(\text{Re}(s) \geq 0\). We assume \(p_{k} = \lambda_{\text{PG}}^{k}E(C_{\text{PG}}^{k}) = \lambda_{\text{PG}}^{k}m_{F} < 1\) for all \(1 \leq k \leq m_{\text{PG}}\), so that for each \(1 \leq k \leq m_{\text{PG}}\), the steady-state delay time of Group-\(k\) PG packet denoted by \(D_{\text{PG}}^{k}\) is governed by a unique distribution. Let \(G_{D_{\text{PG}}^{k}}(s)\) denote the Laplace–Stieltjes transform of the steady-state distribution function for \(D_{\text{PG}}^{k}\). Using the embedded Markov chain method described in section 3.1, we obtain

\[
G_{D_{\text{PG}}^{k}}(s) = \frac{1 - \lambda_{\text{PG}}^{k}E(C_{\text{PG}}^{k})}{1 - \lambda_{\text{PG}}^{k}[E(C_{\text{PG}}^{k}) - E(K_{\text{PG}}^{k})]} \times \frac{[\lambda_{\text{PG}}^{k} - s]G_{K_{\text{PG}}^{k}}(s) - \lambda_{\text{PG}}^{k}G_{C_{\text{PG}}^{k}}(s)}{\lambda_{\text{PG}}^{k}[1 - G_{C_{\text{PG}}^{k}}(s)] - s},
\]

for \(1 \leq k \leq m_{\text{PG}}\). Using eq. (13) for \(G_{K_{\text{PG}}^{k}}(s)\) as well as for the calculation of \(E(K_{\text{PG}}^{k})\), and recalling that \(G_{C_{\text{PG}}^{k}}(s) = e^{-m_{F}s}\), \(E(C_{\text{PG}}^{k}) = m_{F}\), we complete eq. (14). We then calculate the moments of the steady-state delay time of Group-\(k\) PG packet, for each \(1 \leq k \leq m_{\text{PG}}\).

4.2. Approximate analysis for channel-allocation packet delay

Depending on the number of slots per frame assigned for the exclusive transmission of CA packets (\(m_{\text{CA}}\)), scheme PG,\(m_{\text{PG}},m_{\text{CA}}\) is divided into two classes: class PG,\(m_{\text{PG}},0\) and class PG,\(m_{\text{PG}},m_{\text{CA}}\) (\(m_{\text{CA}} \geq 1\)). In this section, we consider class PG,\(m_{\text{PG}},0\) as well as class PG,\(m_{\text{PG}},m_{\text{CA}}\) (\(m_{\text{CA}} \geq 1\)). Under scheme PG,\(m_{\text{PG}},0\), a frame consists of \(m_{\text{PG}}\) slots, i.e., \(m_{F} = m_{\text{PG}}\). The \(k\)th slot of each frame is used for the transmission of Group-\(k\) PG packets for \(1 \leq k \leq m_{\text{PG}}\). For scheme PG,\(m_{\text{PG}},m_{\text{CA}}\) (\(m_{\text{CA}} \geq 1\)), we assume the following. The number of PG groups (\(m_{\text{PG}}\)) is an integral-multiple of \(m_{\text{CA}}\), i.e., \(m_{\text{PG}} = (\nu - 1)m_{\text{CA}}\) for \(\nu \in \{2, 3, \ldots\}\). For every \(1 \leq j \leq m_{\text{CA}}\), the \((\nu j)\)th slot in each frame is used for the exclusive transmission of a
CA packets. In this manner, the slots exclusively assigned for CA packets are uniformly distributed over the frame. Group-k PG packets are served during the k'th slot in each frame, where k' is defined as follows. For 1 \leq k \leq m_{PG}, set j' = \max\{j: j(\nu - 1) < k, 0 \leq j \leq m_{CA} - 1\} and i' = k - j'(\nu - 1). Then, k' = i' + j'\nu. (See figure 2, where 6 PG groups are established and m_{CA} = 2.)

In order to obtain the statistical properties of the CA packet delay time, we present an approximation method for calculating the distribution function of the CA packet completion time. Let \{R_{n}, n \geq 1\} denote the departure point process of CA packets from the BS. Let \{R'_{l}, l \geq 1\} be a subsequence of \{R_{n}, n \geq 1\} such that X_{R'_{l}+} = 0. Note that at time R'_{l}, the completion time of a CA packet, which is not the first packet to be served during a busy period, starts. Let C_{CA} denote the CA packet completion time which starts at time R'_{l}, and f_{C_{CA}}(m), m \geq 1, be the mass function for C_{CA}. Let \{R''_{l}, l \geq 1\} be a subsequence of \{R_{n}, n \geq 1\} such that X_{R''_{l}+} = 0. Thus,

\[ \{R_{n}, n \geq 1\} = \{R'_{l}, l \geq 1\} \cup \{R''_{l}, l \geq 1\}. \]

Define A''_{l+1} to be the first CA packet arrival time after R''_{l}, for l \geq 1. At time A''_{l}, the completion time of a CA packet, which is the first packet to be served during a busy period, starts. Let K_{CA} denote the CA packet completion time which starts at A''_{l}, and F_{K_{CA}}(x) be the distribution function for K_{CA}. Recalling that packet arrivals follow a continuous-time Poisson process, we set T_{l} = \min\{m: m \geq A''_{l}, m = 1, 2, \ldots\} and U = T_{l} - A''.

The random variable U represents the length of the time interval between the CA packet arrival time A''_{l} and the start time of the slot following the packet arrival, i.e., the arrival slot latency. Let F_{U}(x) denote the distribution function of U. Since A''_{l} - R''_{l-1} is the length of an exponentially distributed idle period, we obtain

\[
F_{U}(x) = 0 \cdot I_{(x<0)} + e^{-\lambda_{CA}}[e^{\lambda_{CA}x} - 1]I_{(0 \leq x < 1)} + 1 \cdot I_{(x \geq 1)},
\]

We express K_{CA} as K_{CA} = U + V, where V is a random variable which takes positive integer values (figure 3). Let f_{V}(m), m \geq 1, denote the mass function for V. For m \geq 1, we define \(\alpha(m) = \max\{n: nm_{F} < m, n = 0, 1, \ldots\}\) and \(\beta(m) = m - \alpha(m)m_{F}\). Note that \(\alpha(m) \geq 0, \beta(m) \in \{1, \ldots, m_{F}\}\) and \(m = \beta(m) + \alpha(m)m_{F}\) for all \(m \geq 1\). We observe that \(\alpha(m) + 1\) represents the index of the frame in which the mth slot resides, while \(\beta(m)\) designates the position of the mth slot in its frame. Define \(\psi_{R}(k) = P(\beta(R_{n}) = k)\) and \(\psi_{T}(k) = P(\beta(T_{l}) = k), \) for \(1 \leq k \leq m_{F}\).

Thus, \(\psi_{R}(k)\) represents the probability that a CA packet departure takes place at the kth slot of a frame, while \(\psi_{T}(k)\) is the probability that a busy period associated with the process \(X_{CA}\) starts during the kth slot of a frame. In the following, we develop a method for the calculation of \(f_{C_{CA}}(m)\) and \(F_{K_{CA}}(x)\), under schemes PG, m_{PG},0 and PG,m_{PG},m_{CA} (m_{CA} \geq 1), respectively.

Under scheme PG,m_{PG},m_{CA}, CA packet completion times depend on the evolution of the processes \(\{A'', 1 \leq k \leq m_{PG}\}\). For this reason, we investigate the following discrete-time system-size process.

**Discrete-time PG packet system-size process under scheme PG,m_{PG},m_{CA}**

Suppose that in each frame, the k'th slot (1 \leq k \leq m_{PG}) is used for the transmission of a PG packet destined to a MS which belongs to Group-k (1 \leq k \leq m_{PG}). For each 1 \leq k \leq m_{PG}, define \(Y_{n}^{k} = X_{nm_{F}+k-1}^{k}\) for \(n \geq 0\). Note that \(Y_{n}^{k}\) represents the system-size of Group-k PG packets at the start of that group’s paging slot during the \((n+1)\)st frame. Set \(p_{k} = \lambda_{PG}^{k}m_{F}\) for \(1 \leq k \leq m_{PG}\). Then, for each 1 \leq k \leq m_{PG}, \(Y_{n}^{k} = \{Y_{n}^{k}, n \geq 0\}\) is a Markov chain on the state space \(S = \{0, 1, \ldots\}\) with the transition probabilities \(p_{k}(i, j), i, j \in S\) as follows:

\[
p_{k}(0, j) = \frac{e^{-\lambda_{PG}^{k}Y_{n}^{k}j}}{j!} \quad \text{for } j \in S,
\]

\[
p_{k}(i, j) = \frac{e^{-\lambda_{PG}^{k}Y_{n}^{k}j-i+1}}{(y-x+1)!} \quad \text{for } j \geq i - 1, i \geq 1.
\]

Hereafter, we assume \(p_{k} < 1\) for all 1 \leq k \leq m_{PG}, so that \(Y_{n}^{k}\) is an irreducible and positive-recurrent Markov chain for each 1 \leq k \leq m_{PG}. Let \(p_{k}^{(n)}(i, j), i, j \in S\) denote
the n-step transition probabilities for $h^k$ and set $\phi_k(i) = p^n_k(i, 0), n \geq i, i \in S$. Then, we obtain

$$\phi_k^i(n) = 0,$$

for $n \leq i - 1$, and

$$\phi_k^i(n) = e^{-i/n} 1 + \sum_{m=1}^{n-i} \left( \frac{n-m}{m!} \right) \left( \frac{m-1}{(m-1)!} \right) \left[ \frac{n}{m} \right].$$

for $n \geq i$. Let $\{ h_{Y^k}(i), i \in S \}$ denote the steady-state distribution for $h^k$ and $Y^k$ as random variables, governed by $\{ h_{Y^k}(i), i \in S \}$, so that $Y_n \xrightarrow{d} Y^k$ as $n \to \infty$, for $1 \leq k \leq m_{PG}$. Note that $h_{Y_k}(0) = 1 - \rho_k$. Define

$$h_k(m) = P(Y_m = 0, Y_{m-1} \geq 1, \ldots, Y_{k+1} \geq 1 | Y_0 = i)$$

for $m \geq 1, 1 \leq k \leq m_{PG}$ and $i \in S$. The function $h_k(m)$ represents the probability that the first hitting time of state 0 by $Y_n$: $Y_0 = i, n \geq 1$ is equal to $m$. Using eq. (16), we obtain

$$h_k^i(1) = \phi_k^i(1),$$

$$h_k^i(m) = \phi_k^i(m) - \sum_{l=1}^{m-1} \phi_k^l(l)h_k^i(m-l),$$

for $m \geq 2$. Set

$$h^k(m) = E(h_{Y^k}(m)) = \sum_{i=0}^{\infty} h_k(m) f_{Y^k}(i).$$

We note that $h^k(m)$ represents the probability that at steady-state the Group-k PG packet system-size will transit into state 0 in $m$ frames. Then, from eq. (17), we have

$$h_k^i(1) = 1 - \rho_k,$$

$$h_k^i(m) = (1 - \rho_k) - \sum_{l=1}^{m-1} \phi_k^l(l)h_k^i(m-l),$$

for $m \geq 2$. For later use, we set $H_k^i(m) = 1 - \sum_{l=1}^{m} h_k^i(l)$ and $H_k^i(m) = 1 - \sum_{l=1}^{m} h^i(l)$, for $m \geq 1, 1 \leq k \leq m_{PG}$ and $i \in S$. 

CA packet completion time under scheme PG.m_{PG,0}. Under scheme PG.m_{PG,0}, Group-k PG packets are served during the kth slot in each frame. First, consider the nominal CA packet completion time which starts at $R'_i$ (C_{CA}). This packet departs at the end of the first idle paging slot after $R'_i$. (We say a paging slot is idle, if at the start of the slot, the corresponding PG packet system-size is 0.) Let $Q((k_0, m_{PG,0}), (k, n))$ denote the conditional probability

$$P(C_{CA} = k + nm_{PG}, R'_i = k_0) = P((k_0, m_{PG,0}), (k, n))$$

for $1 \leq k_0, k \leq m_{PG}$ and $n_0 \geq 0$. For $k_0 = 0$, i.e., when the completion time starts at the beginning of a frame, the conditional probability is written as the following product form. Note that the kth slot of the $(n + n_0 + 2)$nd frame is the first idle paging slot after $m_{PG} + n_0 m_{PG}$ (figure 4).

$$Q((m_{PG}, n_0), (k, n)) = \prod_{r=1}^{k-1} P(Y_{n_0+n} = 0, Y_{n_0+n+1} \geq 1, \ldots, Y_{n_0+m} \geq 1 | A)$$

$$\times \prod_{r=k+1}^{m_{PG}} P(Y_{n_0+n} \geq 1, \ldots, Y_{n_0+n+m} \geq 1 | A)$$

$$= \prod_{r=1}^{k-1} \sum_{i=0}^{R'_i(n)} H'_i(n)P(Y_{n_0} = i | A)$$

$$\sum_{i=0}^{R'_i(n)} H'_i(n-1)P(Y_{n_0} = i | A)$$

where $A$ is the event $\{ R'_i = m_{PG} + n_0 m_{PG} \}$ and the function $h_k(n)$ is defined in (17). For $1 \leq k_0 \leq m_{PG} - 1$, the conditional probability $Q((k_0, n_0), (k, n))$ is presented in appendix. We make the following two assumptions. First, we set given $R'_i = k_0 + n_0 m_{PG}$,

$$Y_{n_0+k} \xrightarrow{d} Y^k_{PG},$$

for $1 \leq k \leq m_{PG}$. Thus, we assume that Group-k PG packet system-size at the start of the kth slot in the frame preceding the completion start time is characterized by the steady-state distribution of the corresponding PG packet system-size, for each $1 \leq k \leq m_{PG}$. Note that under assumption (20), $P(Y_{n_0+k-1} = i | R'_i = k_0 + n_0 m_{PG}) = P(Y_{PG} = i) = f_{Y_k}(i)$, for all $1 \leq k \leq m_{PG}$. Therefore, for $k_0 = m_{PG}$, we obtain

$$Q((m_{PG}, n_0), (k, n)) = \prod_{r=1}^{k-1} H'_i(n)h^k(n) \prod_{r=k+1}^{m_{PG}} H'_i(n-1).$$

For $1 \leq k_0 \leq m_{PG} - 1$, the conditional probability $Q((k_0, n_0), (k, n))$ under assumption (20) is presented in appendix. Note that under assumption (20), $Q((k_0, n_0), (k, n))$ does not depend on the frame index of the completion start time.
for $1 \leq k \leq m_F$ and $n \geq 0$. Note that we calculate the
function $\psi_T(k_0)$ by using eqs. (22) and (24). Since $K_{CA} = U + V$, we conclude the distribution function for $K_{CA}$ to be given by

$$F_{K_{CA}}(x) = \sum_{m=1}^{\infty} F_U(x - m) f_V(m),$$

(27)

where $F_U(x)$ and $f_V(m)$ are given in (15) and (26). Note that under assumptions (22) and (25), for every CA packet which is the first packet to be served in a busy period, its completion time is characterized by the distribution function $F_{K_{CA}}(x)$ given in (27).

CA packet completion time under scheme PG,m_{PG},m_{CA} ($m_{CA} \geq 1$). First, we consider the nominal CA packet completion time which starts at $R'_i$ ($K_{CA}$). Recall that $\nu = m_F/m_{CA} = (m_{PG} + m_{CA})/m_{CA} \in [2,3,\ldots]$. Under scheme PG,m_{PG},m_{CA} ($m_{CA} \geq 1$), the CA packet whose completion time starts at $R'_i$ departs at the end of the first idle paging slot after $R'_i$, if an idle paging slot following $R'_i$ precedes the first exclusively assigned slot for CA packets. Otherwise, the CA packet departs at the end of the first exclusively assigned slot for CA packets. Thus, under scheme PG,m_{PG},m_{CA} ($m_{CA} \geq 1$), $C_{CA}$ is bounded by $\nu$. Let $Q((i_0 + j\nu, n_0), m)$ denote the conditional probability $P(C_{CA} = m \mid R'_i = [i_0 + j\nu] + n_0 m_F)$, for $1 \leq i_0 \leq \nu$, $0 \leq j_0 \leq m_{CA} - 1$, $n_0 \geq 1$ and $1 \leq m \leq \nu$. For $i_0 + j\nu = m_F$, i.e., when the completion time starts at the beginning of a frame, the conditional probability is written as follows (figure 5):

$$Q((m_F, n_0), m)$$

$$= \prod_{r=1}^{m-1} P(Y_{n_0+1}^r \geq 1 \mid B) P(Y_{n_0+1}^m = 0 \mid B)$$

$$= \prod_{r=1}^{m-1} \sum_{i=0}^{\infty} H_i^r(1)P(Y_{n_0}^r = i \mid B) \times \sum_{i=0}^{\infty} h_i^m(1)P(Y_{n_0}^m = i \mid B),$$

(28)

for $1 \leq k \leq m_F$, where $\theta(k_0, k)$ is an integer in $\{1,\ldots, m_F\}$ such that $\beta(\theta(k_0, k)+k) = k_0$. The mass function of $V$ is calculated by the same approach used to derive eq. (23).

We make an assumption, following the same approximation arguments presented in connection with eq. (20). We assume that given $T_i = k_0 + n_0 m_F$,

$$Y_{n_0-1}^{k_0} = Y_{n_0}^{k_0},$$

(25)

for all $1 \leq k \leq m_F$. Under assumption (25), the conditional probability $P(V = k + n m_F \mid T_i = k_0 + n_0 m_F)$ is equal to $Q((k_0, n_0), (k, n))$ given in eq. (21) and appendix. Replacing $\psi_T(k_0)$ with $\psi_T(k_0)$ in eq. (23), we obtain

$$f_V(k + n m_F) = \sum_{k_i=1}^{m_F} \psi_T(k_0) Q((k_0, n_0), (k, n)),$$

(26)

for $1 \leq k \leq m_F$ and $n \geq 0$. Note that the calculation of the function $\psi_T(k_0)$ by using eqs. (22) and (24). Since $K_{CA} = U + V$, we conclude the distribution function for $K_{CA}$ to be given by

$$F_{K_{CA}}(x) = \sum_{m=1}^{\infty} F_U(x - m) f_V(m),$$

(27)

where $F_U(x)$ and $f_V(m)$ are given in (15) and (26). Note that under assumptions (22) and (25), for every CA packet which is the first packet to be served in a busy period, its completion time is characterized by the distribution function $F_{K_{CA}}(x)$ given in (27).
for $0 \leq m \leq \nu - 1$, where $B$ is the event \{ $R'_i = mF + n_0mF$ \}. Since the $r$th slot is exclusively assigned for the CA packet transmission,

$$Q(mF, n_0, \nu) = \prod_{r=1}^{\nu} \left[ \sum_{i=0}^{\infty} H'_r(1)P(Y'_m = i | B) \right]. \quad (29)$$

For $i_0 + j_0\nu \leq mF - 1$, the conditional probability $Q((i_0 + j_0\nu, n_0, m))$ is presented in appendix. We make the following two assumptions. First, we assume that given $R'_i = [i_0 + j_0\nu] + n_0mF$,

$$Y_k^{n_0-1} = Y_k^{PG}, \quad (30)$$

for $1 \leq k \leq mPG$. Thus, for each $1 \leq k \leq mPG$, $Y_k$ is assumed to have reached steady-state at the start of the $k$'th slot in the frame preceding the completion start time. Since under assumption (30), $P(Y_k^{n_0-1} = i \mid R'_i = [i_0 + j_0\nu] + n_0mF) = P(Y_k^{PG} = i)$, we have

$$Q(mF, n_0, m) = \prod_{r=1}^{\nu-1} \left[ (1 - \rho_r)P_m(1 \leq m \leq \nu - 1) \right] + \prod_{r=1}^{m-1} (1 - \rho_r) P_{mCA}(m = \nu). \quad (31)$$

For $i_0 + j_0\nu \leq mF - 1$, the function $Q((i_0 + j_0\nu, n_0, m))$ under assumption (30) is presented in appendix. Recall that $\psi_R(k) = P(\beta(R_n = x) = k)$ for $1 \leq k \leq mF$. The second assumption is that $\psi_R(k)$ can be expressed as

$$\psi_R(i_0 + j_0\nu) = \frac{1 - \rho_{i_0 + j_0\nu - 1}}{\sum_{k=1}^{mCA} (1 - \rho_k) + mCA}P_m(1 \leq \nu - 1) + \frac{1}{\sum_{k=1}^{mCA} (1 - \rho_k) + mCA}P_{mCA}(mCA = \nu), \quad (32)$$

for $1 \leq j_0 \leq mCA - 1$. By eq. (32), we approximate $\psi_R(k)$ by setting it equal to the probability of a CA packet departure at the $k$th slot of a frame, when the system is overloaded with CA packets. Based on these assumptions, $f_{CA}(m), 1 \leq m \leq \nu$, is calculated as follows:

$$f_{CA}(m) = \sum_{n_0=1}^{mCA-1} \sum_{j_0=1}^{1} P(\alpha(R'_i) = n_0) \times \sum_{i_0=1}^{\nu} \psi_R(i_0 + j_0\nu) \times Q((i_0 + j_0\nu, n_0, m)), \quad (33)$$

where the functions $\psi_R(i_0 + j_0\nu)$ and $Q((i_0 + j_0\nu, n_0, m))$ are given in eqs. (32), (31) and appendix. Note that for every CA packet which is not the first packet to be served in a busy period, its completion time is governed by the mass function $f_{CA}(m)$ given in (33).

Secondly, consider the exceptional CA packet completion time which starts at $N'_F$ ($K_{CA}$). Recall that $K_{CA}$ is expressed as $U + V$. We write the conditional probability $P(V = m \mid T_i = [i_0 + j_0\nu] + n_0mF)$, by replacing $R'_i$ with $T_i$ in the expression for the function $Q((i_0 + j_0\nu, n_0, m))$ given in eqs. (28)–(29) and appendix. Based on the approximation arguments presented in connection with eq. (30), we make the following approximation: Given $T_i = [i_0 + j_0\nu] + n_0mF$,

$$Y_k^{n_0-1} = Y_k^{PG} \quad (34)$$

for all $1 \leq k \leq mPG$. Under assumption (34), the conditional probability $P(V = m \mid T_i = [i_0 + j_0\nu] + n_0mF)$ is equal to $Q((i_0 + j_0\nu, n_0, m))$ given in eq. (31) and appendix. Based on assumptions (32) and (34), $f_V(m)$ is calculated by following the same approach used to derive eq. (33). By the replacement of $\psi_R(k_0)$ with $\psi_T(k_0)$ in eq. (33), we obtain

$$f_V(m) = \sum_{j_0=0}^{mCA-1} \sum_{i_0=1}^{\nu} \psi_T(i_0 + j_0\nu) Q((i_0 + j_0\nu, n_0, m)), \quad (35)$$

for $1 \leq m \leq \nu$, where the function $\psi_T(i_0 + j_0\nu)$ is calculated by using eqs. (24) and (32). Since $K_{CA} = U + V$, we conclude that

$$F_{K_{CA}}(x) = \sum_{m=1}^{\nu} F_U(x - m) f_V(m), \quad (36)$$

where $F_U(x)$ and $f_V(m)$ are given in (15) and (35). Note that under assumptions (32) and (34), for every CA packet which is the first packet to be served in a busy period, its completion time is governed by the distribution function $F_{K_{CA}}(x)$ given in eq. (36).

**CA packet delay time under scheme PG.mPG.mCA.** We have presented an approximation method for the calculation of the distribution functions for CA packet completion times under schemes PG.mPG.0 and PG.mPG.mCA (mCA $\geq 1$), respectively. In association with the CA packet completion time, three assumptions (20), (22) and (25) are made under scheme PG.mPG.0. Under scheme PG.mPG.mCA (mCA $\geq 1$), these assumptions are given in (30), (32) and (34). From these assumptions, we draw the following conclusion. For every CA packet which is the first packet to be served in a busy period, its completion time is governed by the mass function $f_{CA}(m)$, while for other packets, their completion times are identically distributed with the distribution function $F_{K_{CA}}(x)$. Under scheme PG.mPG.0, the functions $f_{CA}(m)$ and $F_{K_{CA}}(x)$ are given in eqs. (23) and (27). Under scheme PG.mPG.mCA (mCA $\geq 1$), these functions are expressed in eqs. (33) and (36). To simplify the analytical procedure, we also assume that the CA packet completion times are statistically independent. Note that such an independent behavior takes place when the system is lightly or heavily loaded. Based on the assumptions on the CA packet completion time, $X_{CA}$ can be regarded as the system-size process of an $M/G/1$ queuing system in which the service time of the first packet in each busy period is governed
by \( F_{KCA}(x) \), while other packets’ service times are characterized by \( f_{C}\).

If \( \rho_{CA} = \lambda_{CA} E(C_{CA}) < 1 \) (as assumed henceforth), then there exists a steady-state distribution for the CA packet delay time, denoted as \( F_{DCA}(x) \). Let \( D_{CA} \) represent a random variable governed by \( F_{DCA}(x) \). By the embedded Markov chain method described in section 3.1, we obtain the Laplace-Stieltjes transform of \( F_{DCA}(x) \) to be given by

\[
G_{DCA}(s) = \frac{1 - \lambda_{CA} E(C_{CA})}{1 - \lambda_{CA} [E(C_{CA}) - E(K_{CA})]} \times \frac{[\lambda_{CA} - s]G_{KCA}(s) - \lambda_{CA} G_{C_{CA}}(s)}{\lambda_{CA} [1 - G_{C_{CA}}(s)] - s}, \tag{37}
\]

where \( G_{C\_CA}(s) \) and \( G_{K\_CA}(s) \) are the Laplace-Stieltjes transforms of the distribution functions for \( C_{CA} \) and \( K_{CA} \), respectively. By differentiating eq. (37) and using eqs. (23) and (27) (eqs. (33) and (36)) to calculate the moments of \( C_{CA} \) and \( K_{CA} \), we obtain the moments of \( D_{CA} \) under scheme PG.0.0 (\( m_{CA} \geq 1 \)).

**Evaluation of analytical results.** In figures 6 and 7, the mean and the standard deviation function of \( D_{CA} \) under scheme PG.0. are shown with respect to the total packet arrival rate \( \lambda \) (\( \lambda = \sum_{k=1}^{m_{PG}} \lambda_{PG} \)). In these figures, we set \( \lambda_{k_{\_PG}} = \lambda_{PG} / m_{PG} \), for all \( 1 \leq k \leq m_{PG} \), where \( \lambda_{PG} = \sum_{k=1}^{m_{PG}} \lambda_{PG} \). These moment functions are compared with simulation results as well as with the results obtained by the degenerated hyperexponential approximation method given in [5]. The shown curves confirm high precision exhibited by our approximation method. Under scheme PG.0.0 (\( m_{CA} \geq 1 \)), the mean and the standard deviation functions of \( D_{CA} \) are evaluated in figures 8 and 9, for \( \gamma = 0.4 \) and \( m_{CA} = 1 \). In these
Dependence of paging and channel-allocation processes. A MS generates a channel-request packet when it is paged, initiates a call or updates its location. Upon receipt of a channel-request, the BS selects a traffic channel and transmits a CA packet to the MS across the forward signaling channel. In this way, a PG packet transmission from the BS induces the transmission of a channel-request by a destined MS which resides in the cell. Upon the completion of a successful transmission of a channel-request packet by the MS, a CA packet is generated at the BS (provided a traffic channel is available for allocation to this connection). Impacted by this process, CA packet arrivals to the BS can statistically depend on PG packet departures from this BS. We have investigated the impact of such dependence effect on the CA packet delay performance through simulation evaluations. In figures 10 and 11, we show performance curves exhibited by the mean CA packet delay time vs. the total traffic load across the shared forward signaling channel. In each figure, we observe that as the fraction of CA packet arrivals induced by PG packet departures relative to the total level of CA packet arrivals increases, the mean CA packet delay also increases. We assume the average channel-request time (representing the average time required by a MS to successfully transmit a channel-request packet) to be equal to 10 and 100 slots, for figures 10 and 11, respectively. Comparing these figures, we conclude that a low channel-request time level invokes higher CA packet delays. This is explained by noting that the fraction of CA packet arrivals which take place during busy periods associated with the PG packet system-size processes is high when the average channel-request time is low. Note, however, that the underlying increases in CA packet delays imposed by the investigated dependencies are normally not significant in the range of loading inducing an acceptable system operation. Hence, the analytical methods developed here can be effectively employed for system design and evaluation purposes.

5. Numerical examples

In sections 3 and 4, we have presented analytic methods for deriving delay characteristics for PG and CA packets under class CA and class PG. The delay levels experienced by packets using these schemes depend on the network parameters: \( \lambda_{k}^{PG} \) and \( \lambda_{CA} \) (arrival rates of Group-\( k \) PG packets and of CA packets), \( m_{PG} \) (the number of PG groups) and \( m_{CA} \) (the number of slots per frame assigned for the exclusive transmission of CA packets). In this section, we present a number of design cases for which we calculate illustrative values for the mean and the standard deviation functions of incurred packet delays, by using the analytic methods described in sections 3 and 4. Note that under class CA, the mean and the standard deviation functions for PG and CA packet delays are calculated exactly. Under class PG, an approximation method is used to obtain those functions for the CA packet delay time, and an exact expression is derived for the mean and standard deviation functions for the PG packet delay time. We demonstrate the impact of these parameters on the packet delay performance and the relative power consumption level required at the handset. Hereafter, we assume that for all \( 1 \leq k_1, k_2 \leq m_{PG}, \lambda_{PG}^{k_1,k_2} = \lambda_{PG}^{k_2,k_1} \), so that a PG packet delay time does not depend on the group to which the destined MS belongs. Let \( \lambda_{PG} = \sum_{k=1}^{m_{PG}} \lambda_{PG}^{k} \). We set \( \gamma = \lambda_{CA}/\lambda \) to denote the relative fraction of CA packets, where \( \lambda \) is the total packet arrival rate, i.e., \( \lambda = \lambda_{PG} + \lambda_{CA} \). Let \( D_{PG} \) and \( D_{CA} \) denote the steady-state packet delay variables for PG and
and CA packets, respectively. The corresponding mean and standard deviation functions are denoted as $\mu_{PG} = E(D_{PG})$, $\mu_{CA} = E(D_{CA})$ and $\sigma_{PG} = \sqrt{\text{Var}(D_{PG})}$, $\sigma_{CA} = \sqrt{\text{Var}(D_{CA})}$. For illustrative purposes, we use $\mu_{PG} + 3\sigma_{PG}$ and $\mu_{CA} + 3\sigma_{CA}$ to indicate the probabilistic peak levels experienced by PG and CA packets.

Figure 12 shows the variation of the peak delay level of the PG packet ($\mu_{PG} + 3\sigma_{PG}$) with respect to $\gamma$ under scheme CA, where we set $\lambda = 0.8$ and $m_{PG} \in \{1, 2, 3, 4\}$. As the number of PG groups ($m_{PG}$) is increased, a better efficiency of power consumption is achieved. In turn, as shown in the figure, PG packets experience then higher delays. Under a prescribed delay limit for PG packets, the selected number of PG groups ($m_{PG}$) can be thus optimized. (Note that under scheme CA, $m_{PG} \neq 0$, the CA packet delay does not depend on $m_{PG}$.) For scheme CA, $m_{PG} = 0$, the formulation of this optimization problem is thus given by:

Given $\lambda$ and $\gamma$,

\[
\begin{align*}
\text{maximize} & \quad m_{PG} \\
\text{subject to} & \quad \mu_{PG} + 3\sigma_{PG} \leq \varepsilon_{PG},
\end{align*}
\]

where $\varepsilon_{PG}$ is the prescribed delay limit for PG packets. For example, set $\lambda = 0.8$ and $\gamma = 0.4$. From figure 12, we obtain the optimal number of PG groups ($m_{PG}$) to be equal to $m_{PG} = 3$, under a delay limit ($\varepsilon_{PG}$) of 40 slots.

Under scheme PG, $m_{PG} = 0$, the impact of $m_{PG}$ on the overall average delay levels ($\mu_{PG} + \mu_{CA}$) is illustrated in figure 13, where $\lambda$ is fixed to 0.8. As shown in figure 13, when $\gamma$ is fixed at 0.1, the overall average delay level is minimized by setting $m_{PG}$ to 3. As noted for $\gamma = 0.1$, an intermediate optimal $m_{PG}$ level may exist. This is explained by noting that for low $m_{PG}$ levels, PG packets (which are granted here higher priority) experience lower delays, while CA packets as a result incur higher delays. The opposite is the case for higher $m_{PG}$ levels (when PG packet delays dominate for lower $\gamma$ values).

In figure 14, we compare schemes CA, $m_{PG} = 0$ and PG, $m_{PG} = 0$ in terms of the performance exhibited by the overall average packet delay function. (In this figure, we set $\lambda = 0.8$.) Figure 14 indicates that the overall average delay level obtained under scheme CA, $m_{PG} = 0$ is better than the one achieved under scheme PG, $m_{PG} = 0$, when $\gamma$ is low and $m_{PG}$ is small. In turn, for higher $m_{PG}$ levels (as well as for higher $\lambda_{CA}$ levels for lower $m_{PG}$ values), the schemes which provide higher priority to PG packets yield better delay performance. Therefore, scheme PG, $m_{PG} = 0$ is preferred under lower handset power consumption level requirements.

Under scheme PG, $m_{PG} = 0$, the peak delay levels of PG and CA packets with respect to $\gamma$ is shown in figures 15 and 16. (In these figures, we set $\lambda = 0.8$. We observe that as the number of PG groups ($m_{PG}$) increases, the peak delay level of the PG packet increases as well, while the peak
Figure 15. Peak level of the PG packet delay under scheme PG.mPG.0 ($\lambda = 0.8$).

Figure 16. Peak level of the CA packet delay under scheme PG.mPG.0 ($\lambda = 0.8$).

Figure 17. Optimal number of PG groups under scheme PG.mPG.0 ($\lambda = 0.8$).

Figure 18. Peak level of the PG packet delay under scheme PG.mPG.1 ($\lambda = 0.8$).

Figure 19. Peak level of the CA packet delay under scheme PG.mPG.1 ($\lambda = 0.8$).

CA packet delay decreases. Hence, under prescribed delay limits for PG and CA packets, an optimal value of $m_{PG}$ can be obtained through the following formulation:

Given $\lambda$ and $\gamma$,

maximize $m_{PG}$

subject to $\mu_{PG} + 3\sigma_{PG} \leq \varepsilon_{PG}$,

$\mu_{CA} + 3\sigma_{CA} \leq \varepsilon_{CA}$,

where $\varepsilon_{PG}$ and $\varepsilon_{CA}$ are prescribed delay limits for PG and CA packets. For example, assume $\lambda = 0.8$. In figure 17, the optimal values of PG groups are presented vs. $\gamma$, where the delay limits ($\varepsilon_{PG}$, $\varepsilon_{CA}$) are assumed to be given by $(15, 15)$, $(15, 40)$, $(40, 15)$ and $(40, 40)$. Note that under ($\varepsilon_{PG}$, $\varepsilon_{CA}$) = $(40, 15)$, there are no feasible PG group levels for $\gamma \in \{0.1, 0.2, 0.3, 0.4\}$. The following conclusions are readily made. Under relaxed delay limits (when ($\varepsilon_{PG}$, $\varepsilon_{CA}$) = $(40, 40)$), more PG groups can be established as the relative loading of PG packets is decreased (and $\gamma$ thus increases). In turn, when tighter PG packet delay limits (such as $(15, 40)$) are imposed, a significant reduction in the feasible values of $m_{PG}$ results (at all $\gamma$ levels). When tighter CA packet delay limits (such as $(40, 15)$) are required, higher $m_{PG}$ levels are used to provide CA packets their lower packet delay objectives.

Figures 18 and 19 show the peak delay levels incurred by PG and CA packets under scheme PG.mPG.1. In these figures, we assume $\lambda = 0.8$. Comparing with the peak delay levels experienced under scheme PG.mPG.0 (as shown in figures 15 and 16), the current scheme, which has an additional slot allocated for the exclusive transmission of CA packets, induces an increase in the peak delay level of a PG packet, and reduces the incurred peak CA packet delay. We also note that the degradation caused in the peak delay level of PG packets is particularly high for lower numbers of PG groups and lower $\gamma$ levels (since then the slot dedicated to CA packets accounts for higher relative channel bandwidth,
I. Rubin, C. Choi / Delay analysis for forward signaling channels

Figure 19. Peak level of the CA packet delay under scheme PG,1
($\lambda = 0.8$).

Figure 20. Optimal number of PG groups under scheme PG,CA,
where ($\varepsilon_{PG,CA} = (40, 15)$ and $\lambda = 0.8$.

and higher paging loads are incurred). Consider the optimization problem mentioned above. When the delay limits for PG and CA packets are set to 40 and 15, respectively, there is no feasible $m_{PG}$ level under scheme PG,0 for lower values of $\gamma$. By employing scheme PG,CA, we can obtain feasible $m_{PG}$ values. Figure 20 exhibits the attainable optimal $m_{PG}$ levels vs. $\gamma$ under scheme PG,CA, when $\lambda$ is fixed to 0.8. We note that for $\gamma \in \{0.3, 0.4\}$, the optimal $m_{PG}$ values are obtained by employing scheme PG,1 (one slot per frame is assigned for the exclusive transmission of CA packets), while scheme PG,0 is employed for $\gamma \in \{0.5, \ldots, 0.9\}$.

6. Conclusions

In this paper, focusing on forward signaling channels used by connection-oriented (such as circuit-switched or ATM) wireless cellular networks, we investigate schemes for multiplexing paging and channel-allocation packets which are transmitted by the cell’s base station to the mobiles. These multiplexing schemes are classified by their channelization plan, access priority assignment and paging group arrangement. The following scheme layouts have been studied and analyzed:

1. Channel-allocation packets are assigned higher priority for transmission across any slot.

2. All the slots of each frame are associated with paging groups. A paging packet destined to a mobile which belongs to Group-$k$ must be then transmitted in the $k$th slot of the frame. Paging packets are provided preference for transmission in their respective slots. Channel-allocation packets can be transmitted at any available slot.

3. Each frame can contain additional slots which are assigned for the exclusive transmission of channel-allocation packets. High access priority is given to paging packets.

For multiplexing schemes in which channel-allocation packets are granted high service priority, we present an exact analytical method for the calculation of the steady-state distributions for paging and channel-allocation packet delay times. For multiplexing schemes under which high priority is given to paging packets, an approximation method is developed to obtain the probabilistic properties of the channel-allocation packet delay. The mean and the standard deviation functions of the channel-allocation packet delay time calculated by the approximation method were shown to yield values which are very close to those obtained by simulation. For this case, exact analysis is carried out for the delay distribution of paging packets.

Using the performance analysis methods derived here, we present a number of examples which illustrate the underlying packet delay and power consumption performance trade-offs. We derive the following conclusions.

1. Under the multiplexing scheme in which channel-allocation packets are granted high priority, the channel-allocation packet delay does not depend on the selected number of paging groups. We can then select the maximum number of groups that guarantees a prescribed delay level for paging packets. A large number of groups is advantageous when it is important to reduce the power consumption at the handset through a “slotted mode” operation.

2. Under the multiplexing scheme in which high priority is given to paging packets, as the number of paging groups increases, the paging packet delay also increases. At the same time, the channel-allocation packet delay decreases. Therefore, to improve power consumption efficiency at the mobile’s handset, the maximum number of groups can be selected in ensur-
ing that the prescribed delay limits for paging packets and for channel-allocation packets are met.

3. Under a prescribed level of power consumption, (equivalently, for a targeted number of paging group levels), we choose a multiplexing scheme which yields the lowest packet delay. We observe that the multiplexing scheme under which high priority is given to channel-allocation packets is superior to the scheme in which paging packets are granted transmission preference, only under loose power consumption requirement levels (when the number of paging groups is low) and under relatively low channel-allocation packet loading level. Otherwise, it is more efficient to grant higher priority to paging packets.

4. Under a tight delay limit for channel-allocation packets, the multiplexing scheme in which all the frame’s slots are dedicated to paging groups and which grants high priority to paging packets, may not yield an acceptable delay level for channel-allocation packets. By adding slots for the exclusive transmission of channel-allocation packets, we can reduce the channel-allocation packet delay to an acceptable level. However, these extra slots lead to an increase in the paging packet delay. Hence, under a tight delay limit for paging packets, the insertion of such slots (for the exclusive transmission of channel-allocation packets) may not yield a feasible implementation.

Considering practically implementable priority-based schemes, the methods and formulas derived here can be employed to choose the most desirable operating structure of the multiplexing algorithm regulating the sharing of the forward signaling channels.

Acknowledgements

This work was supported by Pacific-Bell, Rockwell International and University of California MICRO Grants No. 95-127, 95-128 and 96-157.

Appendix

A. Derivation of \( \phi(n) \) in section 3.2

Set \( Y_n = X^n_{CA} \) for \( n \geq 0 \). Let \( \{ p^{(n)}(i, j), \ i, j \geq 0 \} \) denote the \( n \)-step transition probabilities for the Markov chain \( Y = \{ Y_n, \ n \geq 0 \} \). Then, we have

\[
p^{(n)}(0, 0) = P(Y_n = 0 \mid Y_0 = 0) = \sum_{k=1}^{n-1} P(Y_n = 0 \mid N((0, n]) = k, Y_0 = 0) \times P(N((0, n]) = k),
\]

where \( N((0, n]) \) is the number of CA packet arrivals in the time-interval \((0, n]\). For each \( k \geq 0 \), we partition the interval \((0, n]\) into two sub-intervals \((0, n-k] \) and \((n-k, n] \), and obtain CA packet arrival patterns in these sub-intervals which yield \( Y_n = 0 \). Note that given \( N((0, n]) = k \), it is necessary that \( N((0, n-k]) \geq 1 \) for \( Y_n = 0 \). Set

\[
a_m = P(Y_n = 0, N((0, n-k]) = k - m \mid N((0, n]) = k, Y_0 = 0)
\]

for \( 0 \leq m \leq k - 1 \). Note that

\[
a_0 = P(N((0, n-k]) = k, N((n-k, n]) = 0 \mid N((0, n]) = k, Y_0 = 0)
\]

and

\[
a_1 = P(N((0, n-k]) = k - 1, N((n-k, n-1]) = 1 \mid N((0, n]) = k, Y_0 = 0)
\]

Thus, we have

\[
a_0 = \left[ \frac{n - k}{n} \right]^k,
\]

\[
a_m = \left( \frac{k}{k-m} \right) \left( \frac{m-1}{nm} \right) \left[ \frac{n-k}{n} \right]^{k-m},
\]

for \( 1 \leq m \leq k - 1 \). Using the expression for \( a_m \) presented above, we calculate the \( n \)-step probability \( p^{(n)}(0, 0) \) as follows:

\[
p^{(n)}(0, 0) = \sum_{k=0}^{n-1} \sum_{m=0}^{k-1} a_m P(N((0, n]) = k)
\]

\[
= P(N((0, n]) = 0)
\]

\[
+ \sum_{k=1}^{n-1} \sum_{m=0}^{k-1} \sum_{m=0}^{k-1} a_m P(N((0, n]) = k)
\]

\[
= e^{-\lambda_{CA} n} \sum_{k=1}^{n-1} \left[ 1 - \frac{k}{n} \right] \frac{e^{-\lambda_{CA}(\lambda_{CA} n)^k}}{k!}
\]

\[
= e^{-\lambda_{CA} n} \left[ 1 + \sum_{k=1}^{n-1} \frac{n^k}{k!} - \frac{n^{k-1}}{(k-1)!} \right] \frac{\lambda_{CA}^k}{n^{k} \lambda_{CA}}.
\]

Using the \( n \)-step transition probability \( p^{(n)}(0, 0) \), we derive the function \( \phi(n) \) as follows:

\[
\phi(n) = p^{(n m_{FP})}(0, 0)
\]

\[
= e^{-\lambda_{CA} n m_{FP}} \times \left[ 1 + \sum_{k=1}^{n m_{FP} - 1} \frac{(n m_{FP})^k}{k!} - \frac{(n m_{FP})^{k-1}}{(k-1)!} \right] \frac{\lambda_{CA}^k}{n^{k} \lambda_{CA}}.
\]

B. Derivation of \( Q(k_0, n_0), (k, n) \) in section 4.2

Set the numbers \( k_1 \) and \( n_1 \) such that \( k_1 + n_1 m_{FP} = [k + n m_{FP}] + [k_0 + n_0 m_{FP}] \). Then, \( k_1 \in \{1, \ldots, m_{FP} \} \) and \( n_1 \in \{1, 2, \ldots \} \). Note that

\[
n_1 - n_0 = n_1 \{ 2 \leq k + k_1 \leq m_{FP} \} + (n + 1) I_{\{m_{FP} + 1 \leq k + k_1 \leq 2 m_{FP} \}}.
\]
Let $A$ denote the event \{\(R'_i = k_0 + n_0m_F\)\}. Then, the conditional probability $Q((k_0, n_0), (k, n))$ is written as the following product form:

$$
Q((k_0, n_0), (k, n)) = \prod_{r=1}^{k_1} P(Y_{r_{n_1}}^r \geq 1, \ldots, Y_{n_0+1}^r \geq 1 \mid A) \times \prod_{r=k_0+1}^{k_1} P(Y_{r_{n_1}}^r \geq 1, \ldots, Y_{n_0}^r \geq 1 \mid A) \times P(Y_{n_1}^k = 0, Y_{n_1-1}^k \geq 1, \ldots, Y_{n_0}^k \geq 1 \mid A) \times \prod_{r=k_0+1}^{m_F} P(Y_{r_{n_1}}^r \geq 1, \ldots, Y_{n_0}^r \geq 1 \mid A)
$$

$$
= \prod_{r=1}^{k_1} \left[ \sum_{i=0}^{\infty} H_i^r(n_1 - n_0)P(Y_{n_0}^r = i \mid A) \right] \times \prod_{r=k_0+1}^{k_1} \left[ \sum_{i=0}^{\infty} H_i^r(n_1 - n_0 + 1)P(Y_{n_0}^r - 1 = i \mid A) \right] \times \sum_{i=0}^{k_1} k_i^k(n_1 - n_0)P(Y_{n_0}^k = i \mid A) \times \prod_{r=k_0+1}^{m_F} \left[ \sum_{i=0}^{\infty} H_i^r(n_1 - n_0)P(Y_{n_0}^r = x \mid A) \right],
$$

for $k_0 \leq k_1 - 1$, and

$$
Q((k_0, n_0), (k, n)) = \prod_{r=1}^{k_1} \left[ \sum_{i=0}^{\infty} H_i^r(n_1 - n_0)P(Y_{n_0}^r = i \mid A) \right] \times \sum_{i=0}^{k_1} k_i^k(n_1 - n_0)P(Y_{n_0}^k = i \mid A) \times \prod_{r=k_1+1}^{\infty} \left[ \sum_{i=0}^{\infty} H_i^r(n_1 - n_0 - 1)P(Y_{n_0}^r = i \mid A) \right] \times \prod_{r=k_0+1}^{m_F} \left[ \sum_{i=0}^{\infty} H_i^r(n_1 - n_0)P(Y_{n_0}^r = i \mid A) \right],
$$

for $k_0 \geq k_1$. Since under assumption (20),

$$
P(Y_{r_{n_1}}^r - 1 = i \mid A) = P(Y_{n_1}^r = i),
$$

for all $1 \leq r \leq m_{PG}$, we obtain

$$
Q((k_0, n_0), (k, n)) = \prod_{r=1}^{k_0} H^r(n_1 - n_0) \prod_{r=k_0+1}^{k_1-1} H^r(n_1 - n_0 + 1) \times k_i^k(n_1 - n_0 + 1) \prod_{r=k_1+1}^{m_F} H^r(n_1 - n_0),
$$

for $k_0 \leq k_1 - 1$, and

$$
Q((k_0, n_0), (k, n)) = \prod_{r=1}^{k_1-1} H^r(n_1 - n_0)h_i^k(n_1 - n_0) \times \prod_{r=k_0+1}^{k_1} H^r(n_1 - n_0 - 1) \prod_{r=k_0+1}^{m_F} H^r(n_1 - n_0),
$$

for $k_0 \geq k_1$.

**C. Derivation of $Q((i_0 + j_0\nu, n_0), m)$ in section 4.2**

Let $B$ denote the event \{\(R'_i = [i_0 + j_0\nu] + n_0m_F\)\}. Note that the completion time starts at the beginning of the \((i_0 + j_0\nu + 1)\)st slot of the \((n_0 + 1)\)st frame. We derive the conditional probability $Q((i_0 + j_0\nu, n_0), m)$ for each of all possible values for $i_0$ and $j_0$.

**Case 1:** $1 \leq i_0 \leq \nu - 2$ and $0 \leq j_0 \leq m_{CA} - 1$. The \((i_0 + j_0\nu + 1)\)st slot is used for PG Group \((i_0 + j_0\nu - 1)\)th, and \((\nu + j_0\nu)\)th slot is exclusively assigned for the CA packet transmission. Set $k_0 = i_0 + j_0(\nu - 1)$. The conditional probability is written as follows:

$$
Q((i_0 + j_0\nu, n_0), m) = \prod_{r=k_0+1}^{k_0+m-1} P(Y^r_{n_0} \geq 1 \mid B)P(Y^m_{k_0+m} = 0 \mid B) \times \prod_{r=k_0+1}^{k_0+m-1} \left[ \sum_{i=0}^{\infty} H_i^r(1)P(Y^r_{n_0} = x \mid B) \right] \times \sum_{i=0}^{k_0+m} k_i^k+m(1)P(Y^k_{k_0+m} = i \mid B),
$$

for $1 \leq m \leq \nu - i_0 - 1$, and

$$
Q((i_0 + j_0\nu, n_0), \nu - i_0) = \prod_{r=k_0+1}^{k_0+\nu-i_0-1} P(Y^r_{n_0} \geq 1 \mid B) \times \prod_{r=k_0+1}^{k_0+\nu-i_0-1} \left[ \sum_{i=0}^{\infty} H_i^r(1)P(Y^r_{n_0} - 1 \geq 1 \mid B) \right].
$$

Under assumption (30), we obtain

$$
Q((i_0 + j_0\nu, n_0), m) = \prod_{r=k_0+1}^{k_0+m-1} (1 - \rho r)\rho k_0+mI_{\{1 \leq m \leq \nu - i_0 - 1\}} \times \prod_{r=k_0+1}^{k_0+m-1} (1 - \rho r)I_{\{m = \nu - i_0\}}.
$$

**Case 2:** $i_0 = \nu - 1$ and $0 \leq j_0 \leq m_{CA} - 1$. The \((i_0 + j_0\nu + 1)\)st slot is exclusively assigned for the CA packet transmission, so that $Q((i_0 + j_0\nu, n_0), 1) = 1$. 

Case 3: \(i_0 = \nu\) and \(0 \leq j_0 \leq m_{\text{CA}} - 2\). The \((i_0 + j_0 \nu + 1)\)st slot is used for PG Group \(((j_0 + 1)(\nu - 1) + 1)\), and the \((i_0 + j_0 \nu)\)th slot is exclusively assigned for the CA packet transmission. Set \(k_0 = i_0 + j_0(\nu - 1)\). The conditional probability is written as
\[
Q(i_0 + j_0 \nu, n_0, m)
= \prod_{r=k_0+1}^{k_0+m-1} \left[ \sum_{i=0}^{\infty} H^r_i(1)P(Y^{r-1}_{n_0-1} = i \mid B) \right]
\]
\[
\times \sum_{i=0}^{\infty} a^{k_0+m}(1)P(Y^{k_0+m}_{n_0-1} = i \mid B),
\]
for \(1 \leq m \leq \nu - 1\), and
\[
Q(i_0 + j_0 \nu, n_0, \nu)
= \prod_{r=k_0+1}^{k_0+\nu-1} \left[ \sum_{i=0}^{\infty} H^r_i(1)P(Y^{r}_{n_0-1} \geq 1 \mid B) \right].
\]
Under assumption (30), we obtain
\[
Q(i_0 + j_0 \nu, n_0, m)
= \prod_{r=k_0+1}^{k_0+m-1} (1 - \rho_r)\rho_{k_0+m}I_{\{1 \leq m \leq \nu - 1\}}
\]
\[
+ \prod_{r=k_0+1}^{k_0+m-1} (1 - \rho_r)I_{\{m=\nu\}}.
\]

References


Izhak Rubin received the B.Sc. and M.Sc. from the Technion – Israel Institute of Technology, Haifa, Israel, in 1964 and 1968, respectively, and the Ph.D. degree from Princeton University, Princeton, NJ, in 1970, all in electrical engineering. During 1964–1967, he served as a Communications Engineer and Officer in the Israeli Signal Corps. In 1967–1968, he worked as an Electronics and C3 Engineer in the Israel Aircraft Industries. Since 1970, he has been on the faculty of the UCLA School of Engineering and Applied Science where he is currently a Professor in the Electrical Engineering Department.

Dr. Rubin has had extensive research, publications, consulting, and industrial experience in the design and analysis of commercial and military computer communications and telecommunications systems and networks. At UCLA, he is leading a large research group. He also serves as President of IRI Computer Communications Corporation, a leading team of computer communications and telecommunications experts engaged in software development and consulting services.

During 1979–1980, he served as Acting Chief Scientist of the Xerox Telecommunications Network. He served as co-chairman of the 1981 IEEE International Symposium on Information Theory; as program chairman of the 1984 NSF-UCLA workshop on Personal Communications; as program chairman for the 1987 IEEE INFOCOM conference; and as program co-chair of the IEEE 1993 workshop on Local and Metropolitan Area Networks. Dr. Rubin is a Fellow of IEEE, has been serving as editor of the IEEE Transactions on Communications and of the ACM/Baltzer Wireless Networks, and has contributed chapters to texts on telecommunications systems and networks.

Cheon Won Choi was born in Seoul, Korea. He received the B.S. and M.S. in electronics engineering from Seoul National University, Korea, in 1986 and 1988, respectively. He is currently a Ph.D. candidate in electrical engineering at the University of California at Los Angeles. His main research area is the design and analysis of communication networks, especially, wireless networks, and local and metropolitan area networks. Mr. Choi is a member of IEEE.

E-mail: cchoi@ee.ucla.edu.