Synthesis and Throughput Behavior of WDM Meshed-Ring Networks Under Nonuniform Traffic Loading

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Abstract—We study the SMARTNet (Scalable Multichannel Adaptable Ring Terabit Network), an optical wavelength-division packet-switching meshed-ring network employing wavelength routers. We investigate the ways to configure these wavelength routers in adapting to the underlying traffic matrix. One heuristic algorithm for this purpose is developed. We show that for any traffic matrix, the throughput performance of the SMARTNet employing wavelength cross-connect routing devices is upper bounded by that of a corresponding meshed-ring network employing store-and-forward switching devices. The difference diminishes as the number of employed wavelengths increases. Furthermore, for a specific meshed-ring topology, we show that the above upper bound throughput level is achievable under uniform traffic loading at even a small number of wavelengths. Under nonuniform traffic loading, we show that a throughput level equal to 80 to 90% of the upper bound value can be achieved with a small number (five to seven) of wavelengths.

Index Terms—Communication system routing, information rate (throughput), linear programming, networks, topology, wavelength division multiplexing.

I. INTRODUCTION

A N OPTICAL wavelength division packet switching meshed-ring communication network has been introduced recently in [1], [2]. This architecture, also known as SMARTNet (Scalable Multichannel Adaptable Ring Terabit Network), allows the network to offer a significant increase in the throughput capacity in comparison with that realized by other shared-medium networks. The network can be configured to be ATM capable, providing for a seamless interface with wide area BISDN ATM networks. The SMARTNet architecture fits naturally with, and takes best advantage of, the kinds of high speed optoelectronic and optical components that can actually be constructed: very few ports, very simple routing decisions, and little buffering. It follows the trend to ATM and SONET self-healing rings.

Ring topologies have been proven to be highly efficient and survivable network structures, as exemplified by token ring, FDDI, MetaRing [9], and ATMR [10] networks. To explore even higher performance, SMARTNet assumes a meshed-ring physical topology. It is configured as a peripheral ring which connects station access ring interface units (RIU’s) as well as router nodes. The station nodes are interconnected by point-to-point communication links (across the peripheral ring; each such a link is also identified as a ring link). The ring subnetwork is meshed through the use of point-to-point communication links (also identified as chord links) which interconnect the router nodes. This leads to higher network throughput efficiency, and facilitates the segmentation of the network into multiple subnets. Each communication link consists of a pair of fibers. The fibers making each pair carry messages in opposite directions. In addition, through the use of wavelength division multiplexing (WDM), multiple wavelength channels are established across each fiber link. Programmable wavelength-sensitive routers are used. Each router, once programmed, implements a fixed switching matrix. Traffic arriving to a router on an incoming fiber is switched to a pre-designated output fiber (no header reading and processing is required) based on the identity of the input fiber and of the wavelength channel carrying this traffic stream.

The WDM meshed-ring network presented here employs no store-and-forward switching nodes within the network. Depending on the location of the destination station(s), a packet arriving at a source station is assigned a wavelength (which implicitly also designates a route). Hence, queuing delays are incurred by packets only prior to their access into the meshed-ring backbone. Once admitted, packets are transported toward their destination nodes across their designated wavelength highways without incurring any loss or delay jitter penalties. Note that due to the random delays at the switches, the store-and-forward network configuration is difficult to implement optically.

There has been great interest in studying ways to configure optical networks in adapting to the underlying traffic matrix pattern [3]–[6], [11]. Hence, we investigate methods to configure the WDM meshed-ring network in adapting to the underlying offered traffic matrix. We also study the performance feature of such a configuration.

The rest of this paper is organized as follows. Section II describes the network architecture and the system’s operation. In Section III, we define the throughput (information rate) performance index and show that the performance of a meshed-ring employing wavelength cross-connect routers is upper bounded by that of a meshed-ring employing store-and-forward switches. The difference diminishes as the number...
of employed wavelengths increases. An important meshed ring network which employs six router nodes is then studied. For this network, we show in Section IV that this bound is achievable at even a small number of wavelengths. An algorithm for configuring the six router node WDM meshed-ring network is shown and evaluated in Section V. We show that the throughput performance of this meshed-ring network, which uses wavelength cross-connect routers, converges rapidly to that of a corresponding meshed-ring network which employs store-and-forward switches under various nonuniform traffic loading patterns. For example, We demonstrate that by using just five to seven wavelengths, we can achieve a throughput level which is equal from 80 to 90% of the throughput capacity level obtained by employing store-and-forward switches. Conclusions are drawn in Section VI.

II. SYSTEM DESCRIPTION

A. Physical Topology

Consider a meshed ring with \( N \) nodes (also called as ring nodes) and \( K \) wavelength routers (also called as router nodes) located around the ring. The nodes are numbered from 0 to \( N - 1 \) and the routers are denoted as \( R(0), \ldots, R(K - 1) \) [as shown in Fig. 1(a)]. The nodes represent the Ring Interface Units (RIU’s) which serve to connect end-user stations to the meshed ring network. We use directional graph \( G(V, E) \) to represent the physical topology of a meshed ring, where \( V = \{ \text{nodes and routers} \} \) and \( E = \{ \text{ring links and chordal links} \} \).

The \( K \) routers are uniformly located among the network’s \( N \) nodes. It is assumed \( N \) is a multiple integral of \( K \). The degree of router \( R(k) \), denoted as \( D(k) \), represents the router’s number of ports (where each port is actually a port pair serving as an input and output port). For example, \( D(0) = 5 \) and \( D(2) = 4 \) in Fig. 1(a). We assume \( D(k) = D \) for each \( k \) and set \( D = 4 \).

For given values of \( K \) and \( D \), there can be many ways to connect the routers. We consider here symmetric connections. Routers \( R((k + M) \mod K) \) and \( R((k - M) \mod K) \) are said to be the \( M \)th neighboring routers of \( R(k) \) on the ring. For \( D = 4 \), we use \( MR(K, M) \) to represent the meshed-ring network with \( K \) routers and each router is connected by chords to both of its \( M \)th neighboring routers.

We use “\( S \)” to represent a segment of the peripheral ring which lies between two adjacent routers. As shown in Fig. 1(b), segment \( S(k) \) \( (k = 0, \ldots, K - 1) \) is the part of the peripheral ring that lies between routers \( R(k) \) and \( R((k + 1) \mod K) \). We define the digraph \( G_c(V, E_c) \) on the meshed ring as follows: \( V_c = \{ \text{routers} \} \) and \( E_c = \{ \text{segment links and chordal links} \} \); a segment link represents a segment as a single link, ignoring the nodes on the peripheral ring.

B. System Operation

The meshed ring operates as a packet switching network which supports fixed size packets (e.g., ATM cells). A “slotted access” mechanism is used as the multiple-access scheme for every wavelength channel. For each wavelength channel, time slots are established. The time slot duration is equal to the transmission time of a frame. Each frame consists of a header and a payload field which carries the fixed-length packet. Within the header, each time slot is identified as busy or idle. A ready node transmits its packet by inserting it into an idle slot on a preselected wavelength channel (dictated by the underlying routing algorithm to be described later) and converting the slot’s status into a busy one. A destination removal procedure is used, so that a packet is removed by its destination node, which then converts the status of the packet’s slot into idle. In this manner, each wavelength channel’s spatial resources are efficiently reused. See descriptions of the ATM and MetaRing networks for illustrations of spatial reuse ring architectures.

The following are assumed about the network’s operation. 1) Each node employs as many fixed wavelength optical transceivers as required for its multiple access operation 2) Each node has the ability to transmit/receive packets on different wavelength channels simultaneously. 3) All wavelength channels operate at the same data rate.

The structure of a wavelength router is illustrated in Fig. 2(a). Each one of the router’s input ports receives a signal which consists of a number of different wavelengths. These wavelengths are spatially separated through the use of a grating demultiplexer. At each output port, the different wavelengths are multiplexed together through the use of a grating multiplexer. The interconnection pattern of a wavelength router can be described as a wavelength assignment table, as illustrated in Fig. 2(b). For this example, the router is wired so that a signal at wavelength \( \lambda_1 \) arriving at input port \( IP_1 \) is routed to output port \( OP_2 \); a signal at wavelength \( \lambda_2 \) arriving at input port \( IP_1 \) is routed to output port \( OP_3 \); \( \ldots \), and so on.

A wavelength assignment table is defined to be noninterfering if it has no repeating wavelengths in the entries.
of a row or column, as illustrated in Fig. 2(b). This has the following desirable property: each router operates as a cross-connect switch (avoiding the need for store-and-forward header processing and packet switching processing) so that no queuing and buffering delays are incurred at the router and no information loss is experienced inside the network due to congestion bursts. Hence, we require the wavelength assignment table of each router to be a noninterfering one.

We introduce the following class of wavelength walks.

**Definition 1:** For a given configuration of “noninterfering” wavelength assignment tables of the routers, we define a $\lambda$-wavelength walk to be a maximal directional walk on the meshed ring topology such that each node in this walk can transmit packets to its downstream nodes on this walk using wavelength $\lambda$.

We note that if an internodal link belongs to a wavelength walk, then all other links residing on the same ring segment also belong to this wavelength walk (since the walk was defined over the topology). A wavelength walk is identified by 1) its topology: characterized as a directional walk on the meshed ring topology $G_c$; and 2) the wavelength employed. Two wavelength walks are said to be different if either their graph topologies or wavelengths are different. Note that two wavelength walks can utilize the same wavelength if their topologies are directionally chord and segment disjoint. For example, in Fig. 3, wavelength $\lambda_2$ is used to construct six-directional wavelength walks while $\lambda_4$ is used to construct a two-directional wavelength walks.

Different wavelength walks are synthesized by selecting different wavelength assignment tables at the routers. We focus our presentation on the synthesis of wavelength walks.

**Definition 2:** A wavelength path connecting a pair of source-destination nodes is identified by a directional physical path in graph $G$ (a sequence of fiber links, nodes and routers) and a wavelength operating the path.

A wavelength path is therefore described by $\pi(path, \lambda)$; $path$ represents the physical path of the wavelength path and $\lambda$ denotes the wavelength operating the wavelength path.

Given a synthesis of wavelength walks on the network, a wavelength path $\pi(path, \lambda)$ exists if there are wavelength walk(s) $wg(G(wg), \lambda(wg))$ such that $path$ is embedded in $G(wg)$ and $\lambda = \lambda(wg)$.

**III. PERFORMANCE BOUNDS FOR WDM MESHED-RING NETWORKS**

**A. The Throughput Efficiency Measure Definition**

Assume the network is loaded by an arbitrary traffic matrix $T$. The entry $T(s, d)$ represents the traffic rate flowing from node $s$ to node $d$. Moreover, we normalize the traffic matrix $T$ so that $\sum_{s,d} T(s, d) = N^2$, where $N$ denotes the number of network nodes. We study how to synthesize wavelength walks that best satisfy the underlying traffic matrix. We introduce the following definitions.

- $\Delta$ The number of wavelength channels available on the network
- $G_\Delta$ A topology of the wavelength walks synthesized by the $\Delta$ wavelength channels (see, for example, Fig. 3).
- $BW_o(\Delta, G_\Delta)$ The minimum data rate at which each wavelength channel must operate to support the traffic matrix $T$ for a synthesis of wavelength walks $G_\Delta$

We set

$$BW_o(\Delta) = \min_{G_\Delta} BW_o(\Delta, G_\Delta) \tag{1}$$

to represent the minimal value of the wavelength channel data rate, given $\Delta$. We let

$$\eta(\Delta) = \frac{N^2}{2 \cdot \Delta \cdot BW_o(\Delta)} \tag{2}$$

serves as an overall performance measure, representing the network’s throughput efficiency (information rate). It describes the overall carried network throughput per unit network bandwidth. (Note that the factor “2” accounts for the bidirectionality of each link.)

The problem of finding optimal $G_\Delta$ and its $BW_o(\Delta, G_\Delta)$ value can be modeled as a mixed integer linear programming problem, in a way similar to that used in [6]. However, the dimensionality of this problem will then become too large to handle. Therefore, we decompose the synthesis process into two separate procedures: The synthesis of the wavelength walks, and the routing of the offered traffic flows along the selected walks.

**B. An Upper Bound on the Network Throughput Efficiency**

We show that the network throughput efficiency of a WDM meshed-ring network employing wavelength cross-connect routing devices is upper bounded by that of a corresponding meshed-ring network employing conventional store-and-forward switching devices. For the latter, a packet arriving at an input port can be switched to any output port. If the desired output port is not available upon the arrival of a
packet, the packet is queued for later switching. Clearly, store-
and-forward switching offers a potential for better statistical
utilization of the capacity of the network links than that
offered by networking through the sole use of cross-connect
wavelength routing devices. However, we show below that
the difference in the throughput efficiency attainable by these
two different network systems diminishes as the number of
employed wavelengths (Λ) increases.

First we formulate the problem of calculating the network
throughput efficiency under the use of store-and-forward rout-
ing nodes as a minimax programming problem Ms. The fluid flow model is suitable for the ATM packet-switching
environment; it is thus included in the formulation of problem
Ms. We employ the following notations.

\[ I(A) \quad I(A) = 1 \text{ if } A \text{ is true; } I(A) = 0 \text{ if } A \text{ is false.} \]

\[ F_s(l, d) \quad \text{The portion of } F_s(l) \text{ contributed by traffic from source } s \text{ to destination } d. \]

\[ T_s(\pi_{sd}) \quad \text{The fraction of } T(s, d) \text{ that traverses path } \pi_{sd}. \]

The problem Ms is formulated as follows. Given the net-
work topology, and thus the collection of paths \{π_{sd}, ∀(s,d)\},
we wish to distribute the terminal flow across the network, by
assigning to each s → d path \π_{sd} its appropriate flow level
\[ T_s(\pi_{sd}) \text{ to optimize the following measure:} \]

\[ (M_s) \quad BW_s \equiv \min_{T_s} \max_{\pi_{sd}} F_s(l) \]

subject to

\[ = \sum_{(s,d)} T_s(\pi_{sd}) \cdot I(l \in \pi_{sd}) \]

\[ = \sum_{\pi_{sd}} T_s(\pi_{sd}) \cdot I(l \in \pi_{sd}) \]

\[ T_s(\pi_{sd}) \geq 0 \]  

where \[ T_s = \{T_s(\pi_{sd}), \pi_{sd} ∈ Π_{sd}, ∀(s,d)\}, \text{ and } Π_{sd} \text{ is the set of all } s → d \text{ paths derived from the meshed-ring topology} \]

\[ \text{G}. \text{ Therefore, } BW_s \text{ is the minimal level of link flow rate (and thus the minimal value of the required link capacity in each direction), aggregating all } s → d \text{ flows, obtained by splitting each terminal flow across its corresponding paths. We define the network throughput efficiency, under store-and-forward switching, as} \]

\[ η_s = \frac{N^2}{2 \cdot BW_s}. \]

Consider next the network when cross-connect wavelength
router nodes are used. For a given topology of synthesized
wavelength walks \(G_λ\) (employing Λ wavelengths), the prob-
lem of routing the traffic can be formulated as the following
minimax programming problem \(M_λ(G_λ)\). We use the following
notations.

\[ F_s(l, λ) \quad \text{Traffic flow rate on link } l \text{ using wavelength } λ. \]

\[ F_s(l, λ, s, d) \quad \text{Link } l \text{ flow rate at wavelength } λ \text{ contributed by traffic from source } s \text{ to destination } d. \]

\[ Λ_{π_{sd}} \quad \text{The set of all wavelengths which can be used across physical path } π_{sd}. \]

\[ N_λ(π_{sd}) \quad \text{The number of wavelengths which can be used across physical path } π_{sd}; \text{ i.e., } \]

\[ N_λ(π_{sd}) = |Λ_{π_{sd}}|. \]

\[ T_λ(π_{sd}, λ) \quad \text{The fraction of } T(s, d) \text{ that traverses path } π_{sd} \text{ on wavelength } λ. \]

The problem \(M_λ(G_λ)\) is formulated as follows. Given a
network topology \(G\), a specified number of wavelengths (Λ), and a
selected \(G_λ\) with its associated wavelength walk routes and wavelength set \{π_{sd}\}, we select for each wavelength λ and path π_{sd} the fractional flow \[ T_λ(π_{sd}, λ), \text{ to yield the following optimal link flow level per wavelength channel } (BW_λ(Λ,G_λ)): \]

\[ (M_λ(G_λ)) \quad BW_λ(Λ,G_λ) \equiv \min_{T_λ} \max_{λ} F_λ(l, λ) \]

subject to

\[ = \sum_{(s,d)} T_λ(π_{sd}, λ) \cdot I(l \in π_{sd}) \]

\[ = \sum_{π_{sd}} T_λ(π_{sd}, λ) \cdot I(l \in Λ_{π_{sd}}) \]

\[ = \sum_λ T_λ(π_{sd}, λ) \cdot I(λ \in Λ_{π_{sd}}) \]

\[ = \sum_λ T_λ(π_{sd}, λ) \geq 0 \]  

where the third constraint equation indicates that if λ is not a
wavelength member of the set \(Λ_{π_{sd}}\), then \[ T_λ(π_{sd}, λ) = 0. \text{ Note that we assume all wavelength channels to operate at the same data rate; hence, requiring the link capacity of} \]

\[ \max_λ F_λ(l, λ). \text{ The following result indicates that the overall} \]

channel bandwidth (link capacity) required by the WDM cross-
connect network (i.e., Λ, \(BW_λ(Λ)\)) approaches that required
by the store-and-forward network (i.e., \(BW_s\)).

**Theorem 1**: We have \(lim_{Λ→∞} \frac{BW_λ(Λ)}{BW_s} = 1. \)

**Proof**: Assume problem \(M_s\) to yield an optimal solution \(T^*_s(π_{sd})\). Under this solution, the corresponding values for the link flow rates are \{\(F^*_s(l)\)\}. Consider now the WDM network and its flow distribution problem \(M_λ(G_λ)\). We first show that the optimal solution obtained for \(M_s\) can be used to synthesize WDM network with wavelength walks topology \(G_λ\) which yield a feasible flow distribution \(T_λ(π_{sd}, λ)\) for
problem \(M_λ(G_λ)\). Subsequently, the converse is also proved.

Consider the WDM network. Let \(Λ_κ\) denote the minimum number of wavelengths required to synthesize wavelength walks \(G_λ\) in such a way that \(N_λ(π_{sd}) > 0 \text{ if } T^*_s(π_{sd}) > 0. \text{ We consider finite } (K < ∞, N < ∞) \text{ meshed-ring topologies } M_κ(K,M). \text{ For each source-destination node pair } (s,d), \text{ there is a finite number of paths connecting } s \text{ to } d \text{; and the total number of paths, over all } (s,d) \text{ pairs, is also finite. Hence, } \Lambda_κ \text{ must be finite. Assume henceforth that } Λ ≥ Λ_κ. \text{ When } \Lambda = Λ_κ, \text{ we synthesize the wavelength walks topology } G_λ \text{ in such a way that } N_λ(π_{sd}) > 0 \text{ if } T^*_s(π_{sd}) > 0. \text{ Next, we demonstrate a scheme for routing the traffic flows across the meshed-ring network under the wavelength routing operation (i.e., we derive the } \{T_λ(π_{sd})\} \text{ values). We direct all traffic flows across the same physical paths as those used by the optimal solution for } M_s. \text{ (This is made possible since we have synthesized the wavelength walks in such a way that } N_λ(π_{sd}) > 0 \text{ if } T^*_s(π_{sd}) > 0. \text{ Hence} \]

\[ \sum_λ T_λ(π_{sd}, λ) = T^*_s(π_{sd}), \]

Therefore, the flow conservation constraint for \(M_λ(G_λ)\) is automatically satisfied. Moreover, if \(N_λ(π_{sd}) > 1 \text{ and } T^*_s(π_{sd}) > 0\), we set the traffic flow rate \(T^*_s(π_{sd})\) to be uniformly distributed among the wavelengths covering this.
path. That is, we set
\[ T_c(\pi_{sd}, \lambda) = \frac{T_s(\pi_{sd})}{N_\Lambda(\pi_{sd})} \cdot I\{\lambda \in \Lambda_{\pi_{sd}}\}. \]  
We then obtain
\[ F_0(l, \lambda) = \sum_{(s,d)} \sum_{\pi_{sd}} \frac{T_s(\pi_{sd})}{N_\Lambda(\pi_{sd})} \cdot I\{\lambda \in \Lambda_{\pi_{sd}}\} \cdot I\{l \in \pi_{sd}\}. \]
Hence
\[ F_0(l, \lambda) = \sum_{(s,d)} \sum_{\pi_{sd}} I\{l \in \pi_{sd}\}. \]
where \( F_0 \) is defined as the corresponding second term above.

When \( \Lambda = x \cdot \Lambda_\ell \), where \( x \) is an integer, we synthesize the wavelength walks topology \( G_\Lambda \) in such a way that \( N_\Lambda(\pi_{sd}) = x \cdot N_{\Lambda_\ell}(\pi_{sd}) \). That is, we scale up by a fixed factor the number of wavelength walks by replicating those synthesized above by using \( \Lambda_\ell \) wavelengths. In this manner, we obtain \( N_\Lambda(\pi_{sd}) \rightarrow \infty \) as \( \Lambda \rightarrow \infty \) for each path \( \pi_{sd} \) and each \( (s,d) \) pair, if \( T_s(\pi_{sd}) > 0 \). Therefore, for every link \( I, e(\Lambda, I) \rightarrow 0 \) as \( \Lambda \rightarrow \infty \). We then obtain
\[ T_s(\pi_{sd}) = \frac{1}{N_\Lambda(\pi_{sd})} - \frac{1}{\Lambda} \cdot I\{l \in \pi_{sd}\} \]
\[ = \frac{F_s(l)}{\Lambda} + \max_{(s,d)} \max_{\pi_{sd}} e(\Lambda, l) \]
where \( e(\Lambda, l) \) is defined as the corresponding second term above.

When \( \Lambda = x \cdot \Lambda_\ell \), where \( x \) is an integer, we synthesize the wavelength walks topology \( G_\Lambda \) in such a way that \( N_\Lambda(\pi_{sd}) = x \cdot N_{\Lambda_\ell}(\pi_{sd}) \). That is, we scale up by a fixed factor the number of wavelength walks by replicating those synthesized above by using \( \Lambda_\ell \) wavelengths. In this manner, we obtain \( N_\Lambda(\pi_{sd}) \rightarrow \infty \) as \( \Lambda \rightarrow \infty \) for each path \( \pi_{sd} \) and each \( (s,d) \) pair, if \( T_s(\pi_{sd}) > 0 \). Therefore, for every link \( I, e(\Lambda, I) \rightarrow 0 \) as \( \Lambda \rightarrow \infty \). We then obtain
\[ \Lambda \cdot BW_c(\Lambda, G_\Lambda) = \max_{(s,d)} \max_{\pi_{sd}} F_s(l, \lambda) \]
\[ = \frac{1}{\Lambda} \max_{(s,d)} \sum_{\pi_{sd}} F_s(l, \lambda) \]
\[ = \frac{1}{\Lambda} \sum_{\pi_{sd}} F_s(l, \lambda) \]
\[ = BW_s. \]
Hence, the optimal solution for problem \( M_s \) should be smaller than that derived from the optimal solution of problem \( M_{\Lambda}(G_\Lambda) \), for every wavelength walks topology \( G_\Lambda \). As a result,
\[ BW_c(\Lambda) \geq BW_s/\Lambda. \]

We can make \( e(\Lambda, l) \) arbitrarily small by selecting \( \Lambda \) to be sufficiently large. Hence, the proof is completed by combining (13) and (15).

The main idea in the proof of Theorem 1 is that 1) we use multiple wavelengths to map the store-and-forward flow distribution to WDM flow distribution and 2) the precision of this mapping of flow rates improves as \( \Lambda \) increases.

IV. PERFORMANCE BEHAVIOR OF A WDM MESHED-RING NETWORK

Under Theorem 1, we have not discussed the underlying rate of convergence of a network’s throughput efficiency measure to the stated upper bound, as \( \Lambda \) increases. In addition, we have not specified the optimal routing scheme which should be employed by the underlying store-and-forward network operation. In the following, we consider the meshed-ring topology MR(6,2). We show that the shortest path routing policy is the optimal routing scheme for the store-and-forward network operation applied to this topology under uniform traffic loading. We also show that for the MR(6,2) topology, \( \eta_\ell(\Lambda) \) is equal to \( \eta_\ell \), for certain \( \Lambda \) levels. Theorem 4 provides a characterization of the rate of convergence of \( \eta_\ell(\Lambda) \) to \( \eta_\ell \).

Theorem 2: Consider the meshed-ring network MR(6,2) loaded with a uniform traffic matrix. The following holds: 1) Under a store-and-forward switching operation, the shortest path routing policy yields the highest throughput efficiency level, \( \eta_\ell(\Lambda) \). 2) Under a wavelength routing operation, we have \( \eta_\ell(\Lambda) = \eta_\ell = 7.2 \), for \( \Lambda = 5\ell \), where \( \ell \) is an integer. For an arbitrary value of \( \Lambda \), we have \( \eta_\ell(\Lambda) \geq \frac{8(2\ell + 1\mod 5)}{2\Lambda} \).

Proof:

a) Store-and-forward switching network operation: We assume the following shortest path routing scheme. For a source-destination pair \((s, d)\), the traffic flow rate \( T(s, d) \) is uniformly distributed among all shortest paths connecting \( s \) to \( d \). Under this routing scheme, we readily observe the followings. The links \( \{l\} \) that have the maximum flow rate \( \{F_s(l)\} \) are those ring links which are adjacent to the routers. We use \( \{l_k\} \) to denote the set of these bottleneck links. There are \( A = 24 \) such bottleneck links (four such links are incident at each one of the routers). Intra-segment flows don’t cross
any bottleneck link and inter-segment flows cross the minimal number of bottleneck links. The resulting $BW_s$ and $\eta_s$ values are $BW_s = \frac{3}{4}(\frac{N^2}{8})^2 = \frac{3}{16}N^2$ and $\eta_s = \frac{N^2}{2BW_s} = 7.2$.

Consider any other routing scheme. Let $BW'_s$ denote the required bandwidth to support the traffic matrix under this routing scheme. We now show that $BW'_s \geq BW_s$. Let $\{T'(\pi_{s,d},d)\}$ represent the corresponding traffic flow rates across the paths connecting a pair $(s,d)$. For a source-destination pair $(s,d)$, if any portion of traffic flow $T(s,d)$ is not directed across a shortest path, then this excess traffic flow traverses routes which cross more bottleneck links than those covered by a shortest path. Hence, this excess flow contributes to link flow rates $\{F_s(l)\}$ of additional bottleneck links. Let $b_{\pi_{s,d}}$ denote the number of bottleneck links contained in path $\pi_{s,d}$.

Then, we have

$$\sum_{\pi_{s,d}} T'_s(\pi_{s,d}) \cdot b_{\pi_{s,d}} \geq \sum_{\pi_{s,d}} T_s(\pi_{s,d}) \cdot b_{\pi_{s,d}} \text{ for all pairs } (s,d),$$

(16)

Hence, we obtain the following:

$$BW'_s \cdot A \geq \sum_i F'_s(b_i)$$

$$= \sum_i \sum_{(s,d)\pi_{s,d}} T'_s(\pi_{s,d}) \cdot I(b_i \in \pi_{s,d})$$

$$= \sum_{(s,d)\pi_{s,d}} T'_s(\pi_{s,d}) \sum_i I(b_i \in \pi_{s,d})$$

$$= \sum_{(s,d)\pi_{s,d}} \left( \sum_i T'_s(\pi_{s,d}) \cdot b_{\pi_{s,d}} \right)$$

$$\geq \sum_{(s,d)\pi_{s,d}} \left( \sum_i T_s(\pi_{s,d}) \cdot b_{\pi_{s,d}} \right)$$

$$= \sum_{(s,d)\pi_{s,d}} F_s(b_i)$$

$$= BW_s \cdot A.$$  

(23)

An inequality is used in (17) since for an alternative routing scheme we may have 1) the bottleneck links may not be the same as those bottleneck links $\{b_i\}$ resulting from the use of the shortest path routing scheme and 2) the flows on bottleneck links $\{b_i\}$ may not all be the same. Equation (18) follows from the definition of $F'_s(b_i)$. Equation (31) follows by applying (16). Finally, (33) holds because all the $A = 24$ bottleneck links $\{b_i\}$ are loaded by the same $F_s(b_i)$ flow value (under the shortest path routing policy). Therefore, $BW'_s \geq BW_s$. As a result, we conclude that the shortest path routing scheme yields the highest throughput efficiency ($\eta_s$) level.

b) Under a wavelength routing operation: Under the shortest path routing scheme described above, the total traffic passing a bottleneck link $b_l$ ($F_s(b_l)$) can be divided into five parts, each of the same value. According to the way the terminal flow with value $F_s(b_l)$ traverses the MR(6,2) topology. If the bottleneck link $b_l$ is an incoming link of a router, then all source-destination pairs $(s,d)$ with a nonzero $F_s(b_l,s,d)$ value must be such that the source $(s)$ nodes reside in the same segment, but the destination $(d)$ nodes reside in different segments. Hence, $F_s(b_l)$ is divided according to the segment in which the $d$ nodes reside in. This is the case shown in Fig. 4(a). On the other hand, if the bottleneck link $b_l$ is an outgoing link of a router, then all $(s,d)$ pair exhibiting a nonzero $F_s(b_l,s,d)$ value must be such that the $d$ nodes reside in the same segment but their $s$ nodes reside in different segments. So $F_s(b_l)$ is divided according to the segment the $s$ nodes reside in. This is illustrated in Fig. 4(b). Incorporating the noninterfering wavelength router constraint, we conclude that $\Delta = 5$ wavelengths are sufficient to implement the shortest wavelength path routing scheme on the meshed-ring network (see [1] and [2] for more details). In this case, we have $BW_o(\Delta = 5) = N^2/72$. As $\Delta = 5 \cdot x$ where $x$ is an integer, we can evenly distribute the link traffic flows ($F_s(b_l)$) on each of the bottleneck links $\{b_i\}$ among these $\Delta$ wavelengths to keep the network throughput efficiency at $\eta_s(\Delta) = 7.2$.

Below we derive a lower bound on the achievable $\eta_s(\Delta)$ level given an arbitrary value for $\Delta$. First we heuristically synthesize a set of wavelength walks. For each group of five wavelengths, they are synthesized in such a way that each $s-d$ flow $T(s,d)$ can follow a shortest path on the meshed-ring by selecting one of these five wavelengths. One such synthesis is shown in Fig. 3. Each of the remaining wavelengths, is synthesized as a peripheral ring (as that used for $\lambda_4$ in Fig. 3). Next we heuristically allocate the traffic flows $\{F_s(b_l,s,d)\}$ among the selected set of wavelength walks. We partition each traffic flow entry $T(s,d)$ into two parts: its first partition, set equal to $y \cdot T(s,d)$, $0 \leq y \leq 1$, follows the shortest paths on the peripheral ring; the other part follows the shortest paths on the meshed-ring.

We readily obtain that the link bandwidth requirement for a ring of $N$ nodes loaded with uniform traffic $T(s,d) = 1, \forall s, d$ is $N^2/8$. We have shown above that $BW_o(\Delta = 5) = N^2/72$. We then derive the partition ($y^* \alpha \beta$) which yields the tightest upper bound on $BW_o(\Delta)$ under such wavelength walk synthesis by solving the following equation [where $\alpha = \frac{3}{4}$ and $\beta = \Delta \alpha + \beta$]:

$$N^2 \cdot \frac{1}{8} \cdot \frac{1}{\beta} \cdot y = N^2 \cdot \frac{1}{72} \cdot \frac{1}{\alpha} \cdot (1-y).$$

As a result, $y^* = \frac{\beta}{5\alpha + \beta}$, which leads to

$$BW_o(\Delta) \leq \frac{N^2}{8 \cdot (9 \cdot \alpha + \beta)}$$

(24)

completing the proof. 

\[ \square \]
The tightness of this bound is examined in the next section.

V. AN ALGORITHM FOR SYNTHESIZING WAVELENGTH WALKS AND NUMERICAL RESULTS

A. Algorithm to Synthesize Wavelength Walks

To use off-the-shelf packages to solve the minimax optimization problems $M_L$ and $M_w(G_{\lambda})$, these problems are transformed to standard linear programming optimization problem formats. The software package GAMS [7] has then been used to obtain optimal solutions for problems $M_L$ and $M_w(G_{\lambda})$. We focus on the synthesis of wavelength walks on the meshed-ring topology MR(6,2). Our methods are readily extended to general meshed-ring topologies.

To ensure connectivity between any source-destination pair of ring nodes, we synthesize here “global” wavelength walks. A “global” walk covers all the segments. Furthermore, we require each such walk to be closed and bidirectional. For example, the wavelength walks synthesized by $\lambda_2$ or $\lambda_3$ in Fig. 3 are two such global bidirectional closed wavelength walks.

Consider the number of different ways to synthesize wavelength walk topology $G_{\lambda}$ so that each configuration can lead to a different $BW_0(A,G_{\lambda})$ value. First, we consider the case $\Lambda = 1$, i.e., a single wavelength walk is used. From the simplicity of the topological structure of MR(6,2), one can readily obtain (through enumeration) that a global bidirectional closed wavelength walk can be synthesized in $B = 120$ different ways. We then have the following property.

Property 1: The number of ways to synthesize global bidirectional closed wavelength walks employing $\Lambda$ wavelengths on MR(6,2) is given by $C_{\Lambda+1}^{B-1}$ where $B = 120$.

Proof: The problem is equivalent to that of finding the number of different ways to allocate $\Lambda$ identical balls into $B$ different bins.

From Property 1, there are a total of 225 150 024 different ways to synthesize global bidirectional closed wavelength walks when $\Lambda = 5$. Since this number is high, the optimal wavelength walk synthesis cannot be performed by exhaustive search. We present the following heuristic greedy algorithm to select good global wavelength walks.

**Global Walk Synthesis Algorithm:**

```
begin
    Construct the 120 global walks $\{r\}$;
    Calculate $\{U(\delta)\}$ from traffic matrix $\{T(s,d)\}$;
    set Num$(\delta)=0$, for each path $\delta$ in $\Delta$;
    for $\Lambda$=1: $\Lambda$, $\Lambda$ odd + $\Lambda$ even do {
        select $\delta$ that maximizes $\sum_{\delta \subseteq \tau} U(\delta) \left( \frac{\Lambda - \text{Num}(\delta)}{\Lambda} \right)$;
        for each path $\delta$ embedded in $\tau$, Num$(\delta) +=$;
    }
end
```

B. Numerical Results and Discussion

We have applied the above algorithm to synthesize wavelength walks on MR(6,2) loaded with a uniform traffic matrix. The resulting performance curves showing the throughput efficiency factor $\eta$ and its dependency on $\Lambda$ are presented in Fig. 6. As expected from Theorem 4, when $\Lambda$ is an integral multiple of five, the algorithm synthesizes wavelength walks in a way that leads to $\eta_{\delta}(\Lambda) = 7.2$. For other values of $\Lambda$, the synthesized wavelength walks may lead to a $\eta_{\delta}(\Lambda)$ value.
higher than the lower bound presented by Theorem 4. This is explained by noting that, in proving Theorem 4, we have synthesized the wavelength walks for $\Delta \mod 5$ wavelengths by using a common peripheral ring. This is generally a suboptimal configuration. In addition, we use a heuristic routing method rather than include optimal rates based on the exact solution of $M_0(G_\lambda)$.

We then consider the following three types of nonuniform traffic matrices.

1) **Random**: For each entry $T(s, d)$, a random number uniformly distributed between zero and one is generated.

2) **Community**: For simplicity, we assume that there are two communities on the network, each of the same size (five nodes). The traffic flow entry between two nodes, both belonging to the same community, is generated as [community factor ratio (CFR) $\times \sigma$], where $\sigma$ is a uniformly distributed random number between zero and one. For the numerical results, we set CFR $= 5$. The traffic rate between two nodes not in the same community is determined by a uniformly distributed random number between zero and one.

3) **Client-Server**: It is assumed that there are two nodes acting as servers and the other nodes represent clients. The two server nodes are randomly selected. It is also assumed that traffic flows only exist between client and server nodes and that the flow matrix is symmetric, i.e., $T(s, c) = T(c, s)$. Half of the traffic generated by a client node destined for one server and the other half flows to the other server. The total traffic generated by a client node is generated as a random number uniformly distributed between zero and one.

We consider MR(6,2) and set $N = 18$. We have considered ten different traffic matrix realizations for each of the three types of traffic matrices mentioned above. The average performance is obtained and presented. We compare the performance under the following three situations: 1) simple ring, 2) meshed-ring with store-and-forward switching operation of the routers, and 3) meshed-ring with wavelength cross-connect operation of the routers. The results are shown in Figs. 7, 8, and 9. From these figures, we observe the following.

1) Applying the “Global Walk Synthesis Algorithm” to the meshed-ring topology generally leads to a higher network throughput efficiency than that obtained by employing a regular ring network. This is induced by the feasibility of synthesizing a larger variety of multiple global walks adapted to the underlying traffic matrix. However, for $\Delta = 1$, for some sample traffic matrices, the meshed-ring topology can exhibit a somewhat lower throughput efficiency level than that obtained by using a regular ring. This is explained by noting that under the heuristic algorithm, we synthesize wavelength walks based solely on aggregate values derived from the whole traffic matrix. Hence, the synthesis is not based on the detailed structure of the traffic matrix, leading to suboptimal selections of wavelength walks. For $\Delta \geq 2$, the topological diversity for selecting wavelength walks is increased. As a result, we have observed the meshed-ring topology to always exhibit a much better throughput efficiency performance than that obtained by using a regular ring.

2) As $\Delta$ increases, $\eta_{2}(\Delta)$ converges to $\eta_{b}$ as expected. The converge rate differs among the different types of traffic. For example, for random traffic, $\eta_{b} = 6.197$, while we achieve a throughput efficiency level of $\eta_{b}(5) = 5.428$, which is equal to 88% of $\eta_{b}$. More noticeably, for client-server type traffic, $\eta_{b} = 3.850$, while $\eta_{b}(5) = 3.793$, which is 98% of $\eta_{b}$! Hence, the “Global Walk Synthesis Algorithm” exhibits excellent performance.
3) The network throughput efficiency from wavelength routing may not monotonically increase with Δ. This is due to the nonuniform random feature of the employed traffic matrix model.

VI. CONCLUSIONS

We studied the network throughput efficiency performance of a meshed-ring optical network under various traffic patterns. The network throughput efficiency of a meshed-ring employing wavelength cross-connect routers is upper bounded by that of a meshed-ring employing store-and-forward switches. As the number of employed wavelengths increases, the difference in throughput efficiency levels attained by these two modes of operation diminishes. In particular, consider the meshed ring network topology MR(6,2) consisting of six routers. Under uniform traffic, the network throughput efficiency levels under these two modes of operation are the same for certain Δ values. Under nonuniform traffic, with a small number of wavelengths (five to seven), the network throughput efficiency attained by employing wavelength cross-connect routers reaches a level equal from 80 to 90% of the throughput upper bound obtained by employing store-and-forward switches.

REFERENCES


